Mesoscopic Physics for Beginners

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GDR Physique Quantique Mésoscopique, Aussois, déc. 2015

Mesoscopic physics = Phase coherence

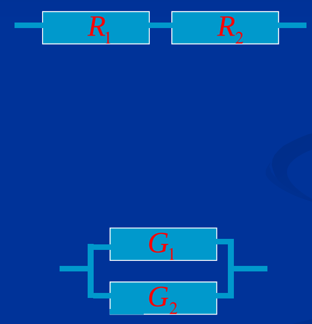
Breakdown of classical laws of electronic transport

$R \neq R_1 + R_2$

$R \neq \rho \frac{L}{S}$

$G \neq G_1 + G_2$

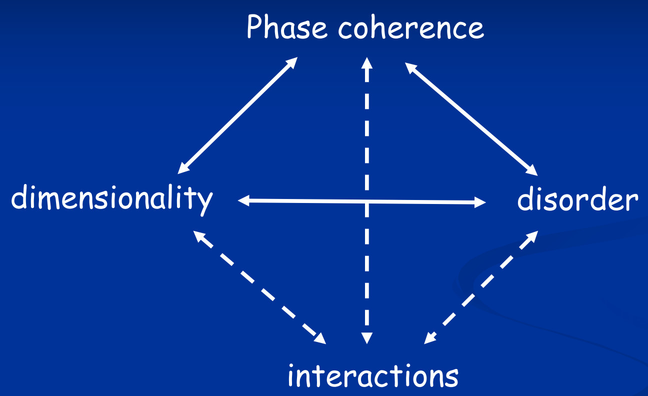
$G \neq \sigma \frac{S}{L}$



$G = \frac{1}{R}$

cf. Two path interferometer…

The mesoscopic triangle



$$\mathcal{H} = \frac{p^2}{2m} + V(\vec{r})$$
$$\psi(\vec{r})$$

$$\mathcal{H} = \hbar c\, \vec{\sigma}.\vec{p} + V(\vec{r})$$
$$\{\psi_A(\vec{r}), \psi_B(\vec{r})\}$$

Domain of mesoscopic physics, deviations to classical transport

Length scales, different regimes

Conduction = transmission Landauer-Buttiker

Quantization of conductance

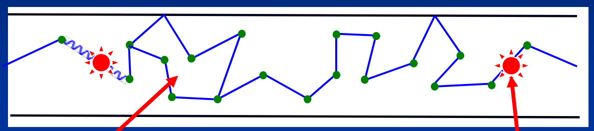
Universal conductance fluctuations

Weak-localization

What limits phase coherence ?

l_e Mean free path : distance between elastic collisions

$\lambda_F \ll l_e$ $l_e = v_F \tau_e$



L_ϕ

interference

Elastic collisions do not break phase coherence

Interaction with an external degree of freedom (phonons, electrons, spin impurities… breaks phase coherence

$L_\phi(T)$ Phase coherence length $L_\phi = \sqrt{D\tau_\varphi}$

Ohm's law $I = GV$ $G = \sigma \frac{S}{L}$

G conductance, σ conductivity

$\sigma = \frac{ne^2\tau_e}{m}$ Drude-Sommerfeld formula

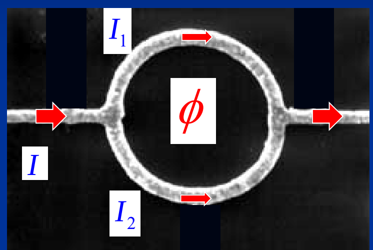
Validity ? Diffusive regime $L \gg l_e$

No quantum effects $L > L_\phi$

R. Webb (IBM, 1985)

THE founding experiment of mesoscopic physics

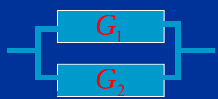
1μm



Classical physics

Ohm's law :

$I = I_1 + I_2$

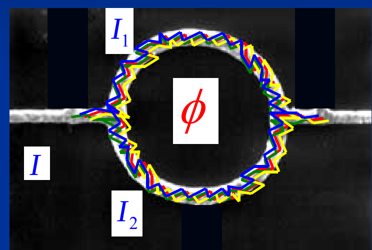


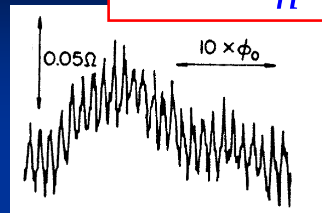
R. Webb (IBM, 1985)

THE founding experiment of mesoscopic physics

$\delta G \sim \frac{e^2}{h} \ll G$

1μm





$\phi_0 = \frac{h}{e}$

$I = I_1 + I_2 + I_{\text{int}} \cos \frac{2\pi\phi}{\phi_0}$

Interferences between electronic waves (cf. Young's slits)

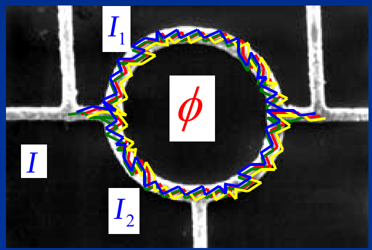
Aharonov-Bohm effect (1959) $l_e \ll L < L_\phi$

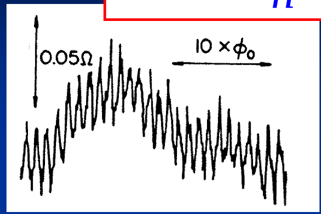
R. Webb (IBM, 1985)

THE founding experiment of mesoscopic phys

$$\delta G \sim \frac{e^2}{h} \ll G$$

1μm





$$\phi_0 = \frac{h}{e}$$

$$I = I_1 + I_2 + I_{\text{int}} \cos \frac{2\pi\phi}{\phi_0}$$

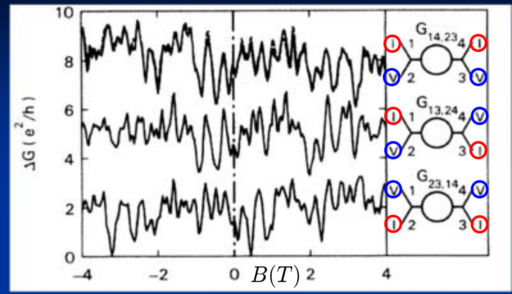
Interferences between electronic waves (cf. Young's slits)

Aharonov-Bohm effect (1959)

$$l_e \ll L < L_\phi$$

$l_e \ll L < L_\phi$

Reproducibles conductance fluctuations



$$\cancel{G = \frac{I}{V}} \rightarrow G_{ij,kl} = \frac{I_{ij}}{V_{kl}}$$

$$\delta G \sim \frac{e^2}{h}$$

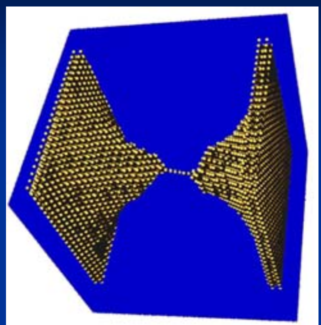
Conductance depends not only on the system to be studied,
but also on its connection to the outside world

Exp. measures a conductance and not a conductivity

Drude-Einstein conductivity provides an average description, valid if $L_\phi < L$

How to go beyond this average and describe interferences, fluctuations ?

What is conductance?



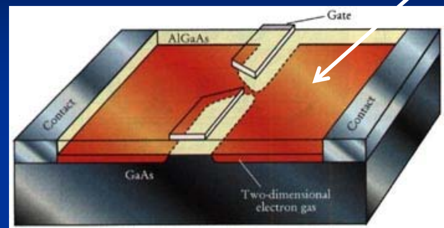
$$G \neq \sigma \frac{S}{L}$$

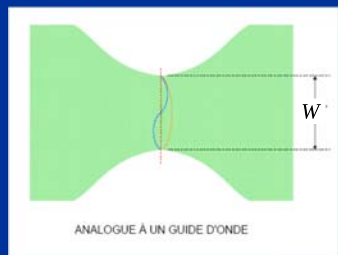
Is the conductance of this Au atomic contact in any way related to the conductivity of gold ?

NO → new concepts, new tools

Quantization of the conductance (1988)

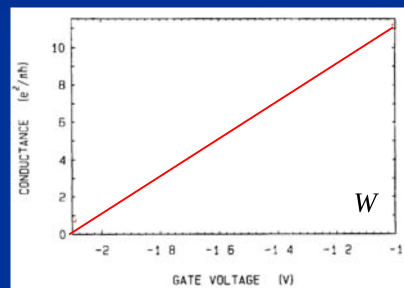
« Quantum Point Contact » QPC ballistic





Classically

$G \propto W$



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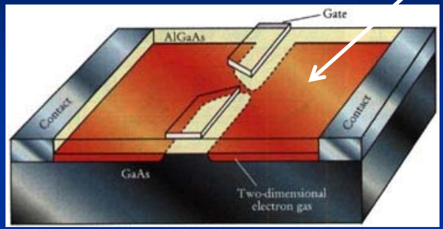
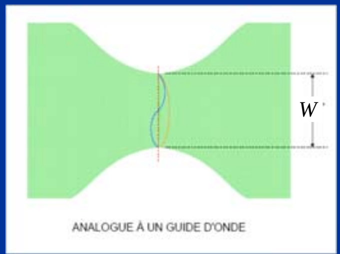
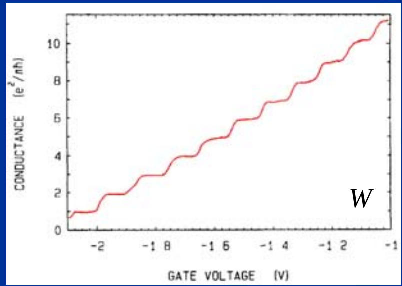


$$G = 2\frac{e^2}{h}M = 2\frac{e^2}{h}\mathrm{int}[\frac{2W}{\lambda_F}]$$





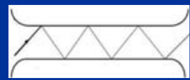
$\frac{e^2}{h}$ Quantum of conductance

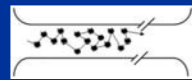
At which scale do we need new concepts ?

nanoscopic | macroscopic

$1nm$ — $10-1000nm$ — $1\mu m$

ballistic l_e diffusive L_ϕ



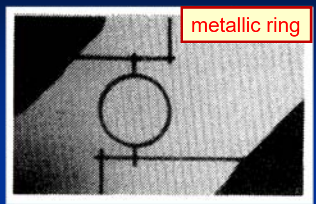


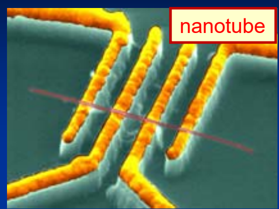
mesoscopic

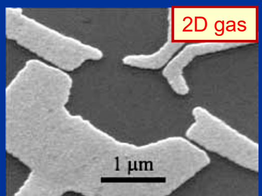
l_e Mean free path : distance between elastic collisions

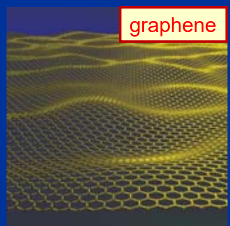
$L_\phi(T)$ Phase coherence length

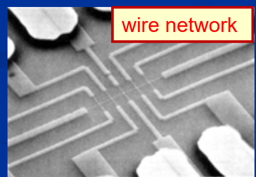
What is conductance?







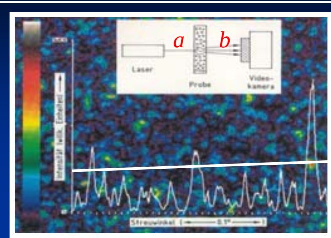




Landauer-Büttiker : conductance = transmission

Conductance – transmission coefficient

Analogies electronics - optics



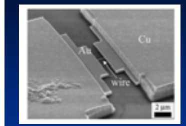
Aharonov-Bohm oscillations ←→ Young's slits

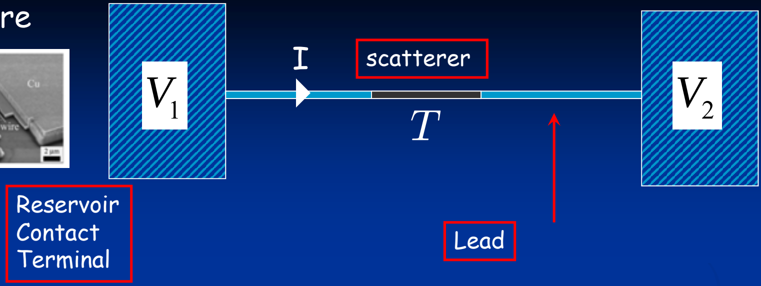
UCF ←→ Speckle

Weak-localization ←→ Coherent backscattering

… electron quantum optics…

1D wire





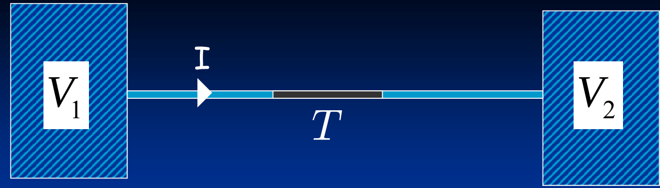
Hypothesis : coherent transport in the wire, dissipation in the reservoirs

- A reservoir absorbs electrons and emits them at its own chemical potential and temperature.
- No phase relation between ingoing and outcoming electrons in a reservoir.
- The scatterer is elastic.
- The resistance of the reservoirs is negligible.

$$I = G(V_1 - V_2)$$

Problem of 1D quantum mechanics

1D wire



Landauer formula

$$G = \frac{2e^2}{h} T$$

$$\frac{e^2}{h} = 1/(25812{,}807\,\Omega)$$

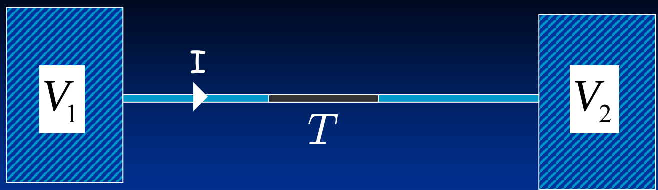
Conductance quantum

Without scatterer

$$G = \frac{2e^2}{h}$$

$$I = G(V_1 - V_2)$$

1D wire



Very qualitatively…

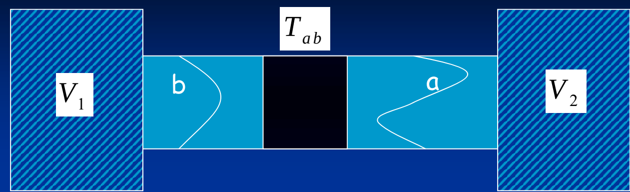
$$I = \frac{charge}{time} = e\,\frac{energy}{h} = e\,\frac{e\Delta V}{h}$$

$$I = \frac{e^2}{h}\Delta V \quad \Rightarrow \quad G = \frac{2e^2}{h}$$

$$time \propto \frac{1}{T} \quad \Rightarrow \quad G = \frac{2e^2}{h} T$$

$$I = G(V_1 - V_2)$$

The «multichannel » case



Courant related to the transmission from a channel 'b' to a channel 'a'

$$I_{ab} = \frac{2e^2}{h} T_{ab}(V_1 - V_2)$$

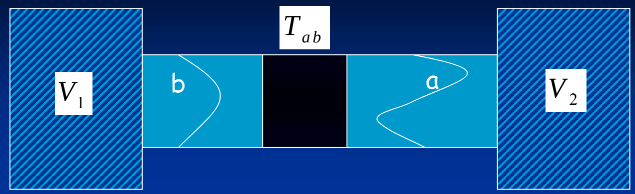
Total current

$$I = \frac{2e^2}{h} \sum_{a,b} T_{ab}(V_1 - V_2)$$

$$G = \frac{2e^2}{h} \sum_{a,b} T_{ab}$$

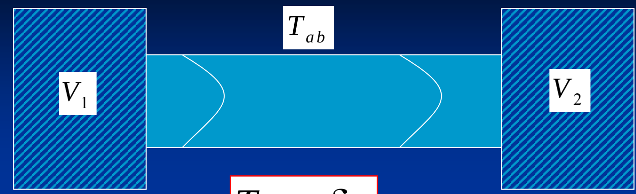
multichannel Landauer formula

The «multichannel » case



$$G = \frac{2e^2}{h} \sum_{a,b} T_{ab}$$

The «multichannel » case



$$T_{ab} = \delta_{ab}$$

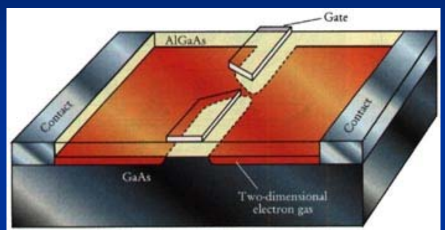
$$G = \frac{2e^2}{h} \sum_{a,b} T_{ab}$$

$$G = 2\frac{e^2}{h} M = 2\frac{e^2}{h} \mathrm{int}[\frac{W}{\lambda_F / 2}]$$

The conductance is proportional to the number of modes transmitted through the wave guide

Quantization of the conductance (1988)

« Quantum Point Contact »



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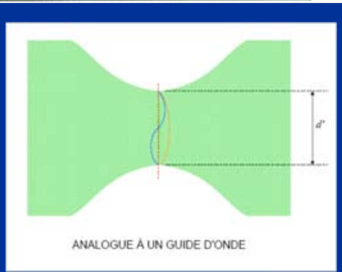
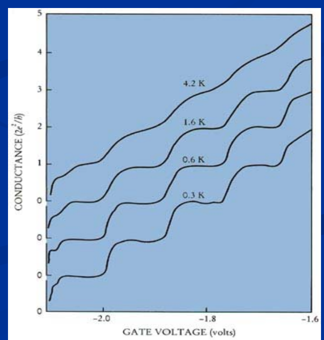
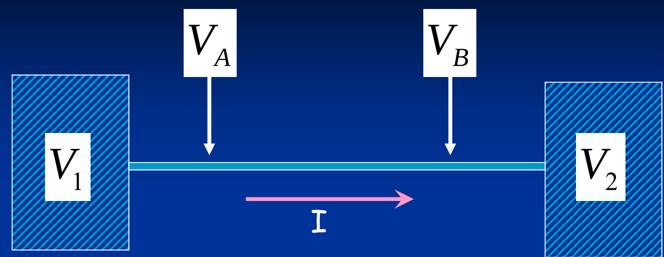




4 vs. 2 terminals



for perfect sample, $V_A = V_B$

$$I = 2\frac{e^2}{h}(V_1 - V_2)$$

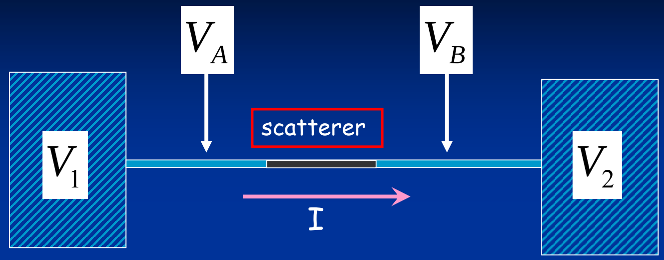
$$G_2 = 2\frac{e^2}{h}$$

no potential drop in the wire :

$$I = (\infty)(V_A - V_B)$$

$$G_4 = \infty$$

4 vs. 2 terminals



with a scatterer $V_A \neq V_B$

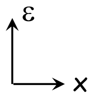
$$I = G_2 (V_1 - V_2)$$

$$I = G_4 (V_A - V_B)$$

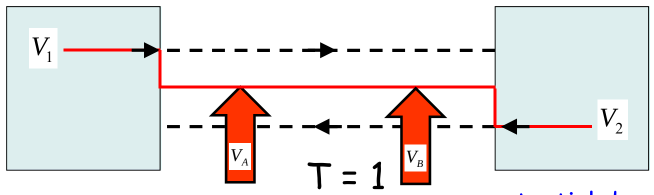
$$G_2 = 2\frac{e^2}{h}T$$

$$G_4 = 2\frac{e^2}{h}\frac{T}{1-T}$$

Potential profile



Ballistic

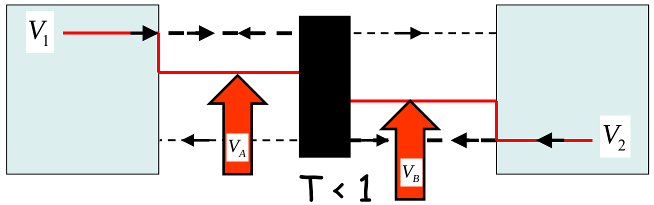


$$G_2 = 2\frac{e^2}{h}$$

$$G_4 = \infty$$

potential drop AT the contacts

One scatterer

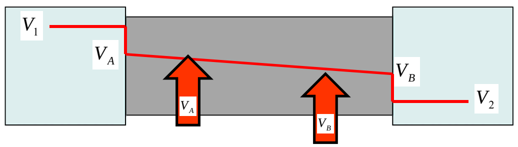


$$G_2 = 2\frac{e^2}{h}T$$

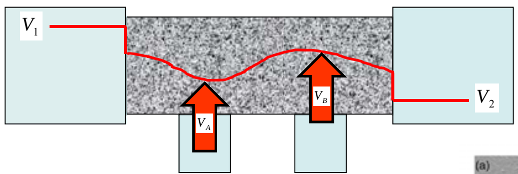
$$G_4 = 2\frac{e^2}{h}\frac{T}{1-T}$$

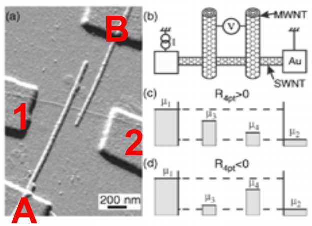
No dissipation in the wire

Diffusive regime



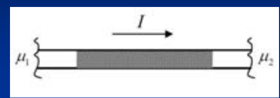
Four terminal disorder + interferences





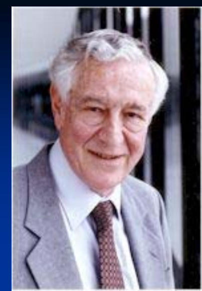
ENS, Paris

Conductance = Transmission

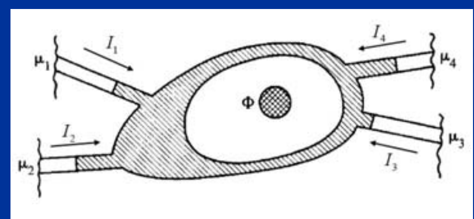


$$G = 2\frac{e^2}{h}T$$

Landauer formula



R. Landauer (1927-1999)

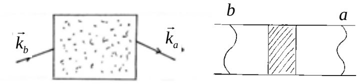


Landauer-Büttiker formalism

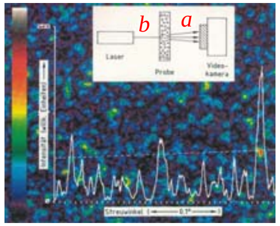
M. Büttiker (1950-2013)

conductance = transmission   analogy with optics

optics - microwaves



T_{ab}

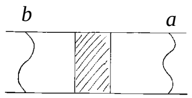


Maret

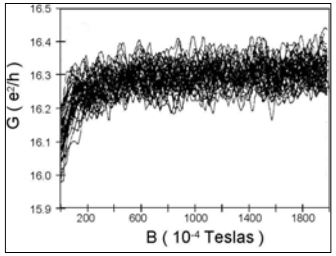
$\delta T_{ab} = \overline{T_{ab}}$

fluctuations ~ average

electronics



$$G = \frac{2e^2}{h} \sum_{a,b} T_{ab}$$



$$\delta G \sim \frac{e^2}{h}$$

$\delta G \ll \overline{G}$

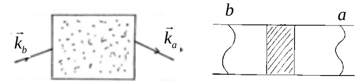
Mailly, Sanquer

???

fluctuations << average

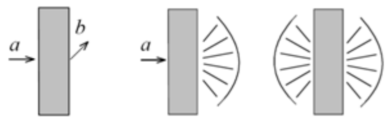
conductance = transmission   analogy with optics

optics - microwaves



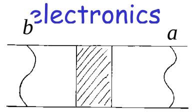
T_{ab}

In optics, you can mesure T_{ab} , T_a , or T



T_{ab} $T_a = \sum_b T_{ab}$ $T = \sum_{a,b} T_{ab}$

electronics



$$G = \frac{2e^2}{h} \sum_{a,b} T_{ab}$$

In electronics, you can only mesure T

$T = \sum_{a,b} T_{ab}$

The fluctuations of $T = \sum_{a,b} T_{ab}$ are much smaller than the fluctuations of T_{ab}

Weak-localization = first quantum correction to classical transport

$$\overline{G} = G_{cl} + \delta G(B)$$

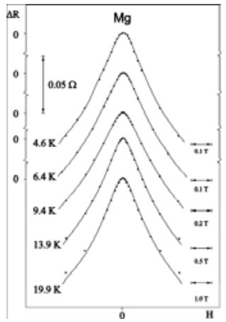
Phase coherence effect

The negative correction is cancelled by a magnetic field B

→ Negative magnetoresistance

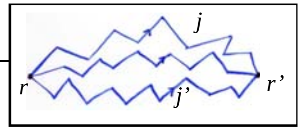
The characteristic field depends on temperature

→ measures the phase coherence



magnetoresistance of a Mg film
Bergmann

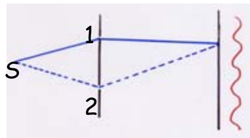
Conductance = Transmission

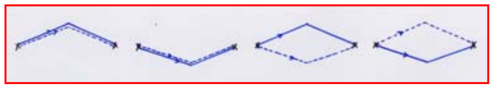


$$G \propto \left| \sum_j A_j(r,r') \right|^2$$

$$G \propto \sum_j \left| A_j(r,r') \right|^2 + \sum_{j,j'} A_j(r,r') A^*_{j'}(r,r')$$

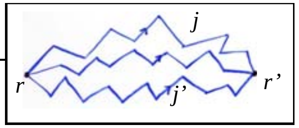
$$I = \left| A_1 + A_2 \right|^2$$





$$I = |A_1|^2 + |A_2|^2 + A_1 A_2^* + A_2 A_1^*$$

Conductance = Transmission



$$G \propto \left|\sum_j A_j(r,r')\right|^2$$

Disorder average

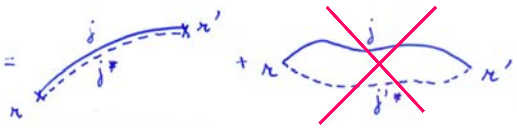
$$\overline{G} \propto \overline{\sum_j \left|A_j(r,r')\right|^2} + \overline{\sum_{j,j'} A_j(r,r')A^*_{j'}(r,r')}$$

Classical term — Interference term

→ Quantum effects

Classical transport: only paired identical trajectoires A_j A_j contribute
If paired trajectories are different, the amplitudes A_j et $A_{j'}$ are different

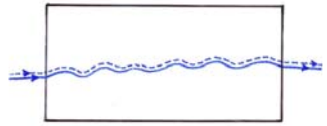
⟹ phases are uncorrelated
⟹ all interference terms disappear in average



???

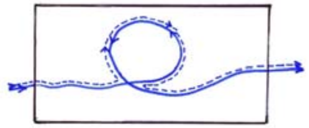
Weak-localization

Classical conductance

G_{cl}  $\propto P_{cl}(0,L)$

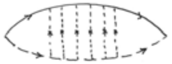
Quantum correction ⟹ **One loop and one crossing**

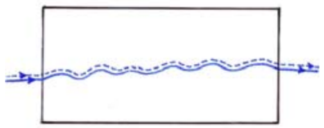
$$\int(-\vec{k})(-d\vec{l}) = \int \vec{k}d\vec{l}$$

ΔG 

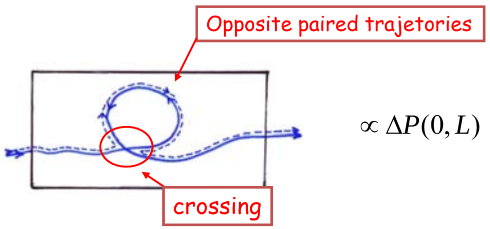
Weak-localization

Classical conductance



G_{cl}  $\propto P_{cl}(0,L)$

Quantum correction ⟹ **One loop and one crossing**

ΔG 

$\propto \Delta P(0,L)$

$P_{\text{int}}(t)$ = **distribution of loops of time t = return probability**

$$\Delta G \sim -\frac{2e^2}{h}\langle P_{\text{int}}(t)\rangle$$

Weak-localization = return probability

$$\Delta G \sim -\frac{2e^2}{h}\langle P_{\text{int}}(t)\rangle = -\frac{2e^2}{h}\int_{\tau_e}^{\tau_D,\tau_\phi} P_{\text{int}}(t)\frac{dt}{\tau_D}$$

$\tau_D = \dfrac{L^2}{D}$ Time spent in the sample

$\tau_\phi = \dfrac{L_\phi^2}{D}$ Phase coherence time

τ_e Elastic collision time

$$\boxed{P_{\text{int}}(t) = \frac{L^d}{(4\pi Dt)^{d/2}}}$$

volume explored after time t

⟹ The return probability $P(t)$ is larger at small d
Phase coherence effects are more important in low dimension

Weak-localization : importance of dimensionality

$$\boxed{\Delta G \propto \int_{\tau_e}^{\tau_\phi} \frac{dt}{t^{d/2}}}$$

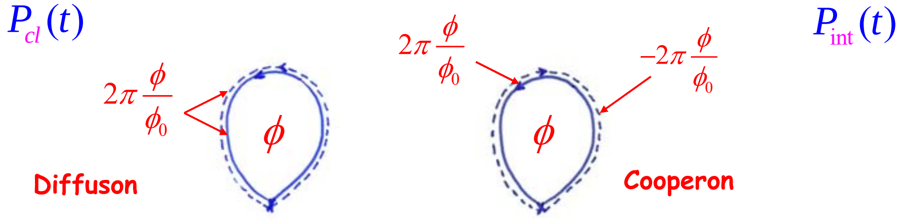
$$\int_{\tau_e}^{\tau_\phi} \frac{dt}{t^{d/2}} \quad \begin{cases} \sqrt{\tau_\phi} - \sqrt{\tau_e} \\ \ln \dfrac{\tau_\phi}{\tau_e} \end{cases}$$

$d = 1$	L_ϕ
$d = 2$	$\ln L_\phi$

The measure of this quantum correction gives access to the phase coherence time (length)

$$L_\phi = \sqrt{D\tau_\phi}$$

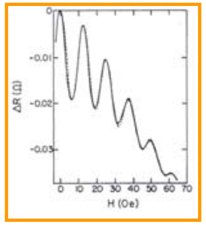
Weak-localization = phase coherence and magnetic field



Cooperon: in a field, time reversed trajectories acquire opposite phases

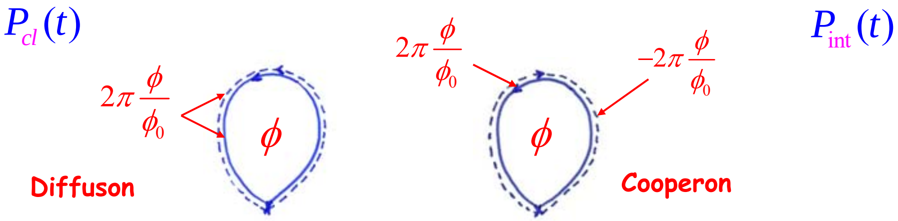
→ **Phase difference** $4\pi \dfrac{\phi}{\phi_0}$ → **oscillations with period** $\dfrac{\phi_0}{2} = \dfrac{h}{2e}$

$$P_{\rm int}(t) = P_{cl}(t)\, e^{4i\pi \frac{\phi}{\phi_0}}$$



Oscillation of the WL correction with the flux
(*Cf. oscillations Sharvin-Sharvin*)

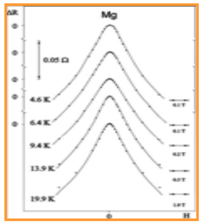
Weak-localization = phase coherence and magnetic field



Cooperon: in a field, time reversed trajectories acquire opposite phases

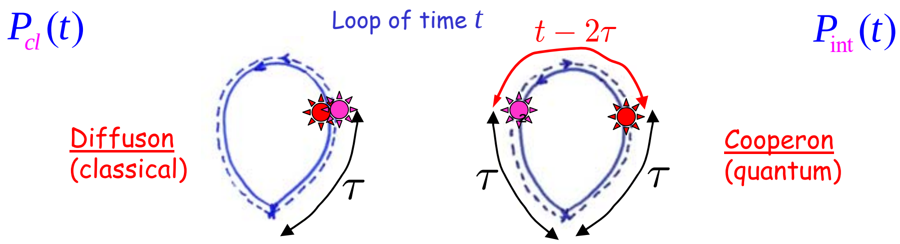
→ **Phase difference** $4\pi \dfrac{\phi}{\phi_0}$ → **oscillations with period** $\dfrac{\phi_0}{2} = \dfrac{h}{2e}$

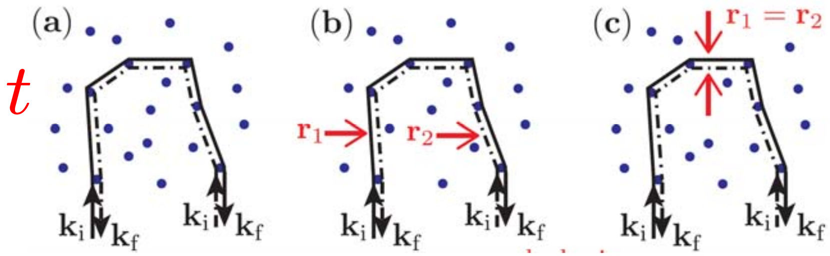
$$P_{\rm int}(t) = P_{cl}(t)\left\langle e^{4i\pi \frac{\phi}{\phi_0}} \right\rangle$$



Negative magnetoresistance
Bergmann

Weak-localization = phase coherence





$$\tau \neq \frac{t}{2} \qquad\qquad \tau = \frac{t}{2}$$

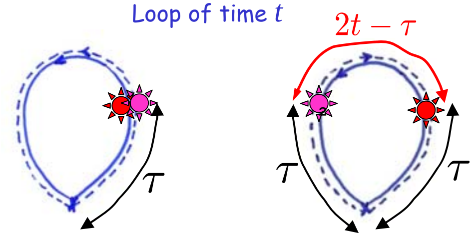
« Suppression » « Revival »

Suppression and revival of WL through control of time-reversal symmetry
Vincent Josse et al. , Institut d'Optique, PRL 2015

Weak-localization = phase coherence

$P_{cl}(t)$ $P_{\text{int}}(t)$

Diffuson (classical)



Cooperon (quantum)

Phase coherence broken after a typical time τ_ϕ
Only trajectories of time $t < \tau_\phi$ contribute to the return probablity and to the WL

$$P_{\text{int}}(t) = P_{cl}(t) e^{-t/\tau_\phi} e^{4i\pi\frac{\phi}{\phi_0}}$$

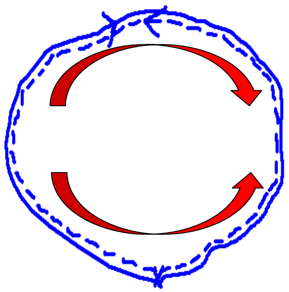
Magnetic impurities, e-e interaction, magnetic impurities

Dephasing

Random dephasing depends on the position of atoms, other electrons, magnetic impurities,…

Dephasing : $4\pi\dfrac{\phi}{\phi_0} + \varphi(t) - \bar{\varphi}(t)$

$$e^{4i\pi\frac{\phi}{\phi_0} + i\Delta\varphi(t)}$$



$2\pi\dfrac{\phi}{\phi_0} + \varphi(t)$

$-2\pi\dfrac{\phi}{\phi_0} + \bar{\varphi}(t)$

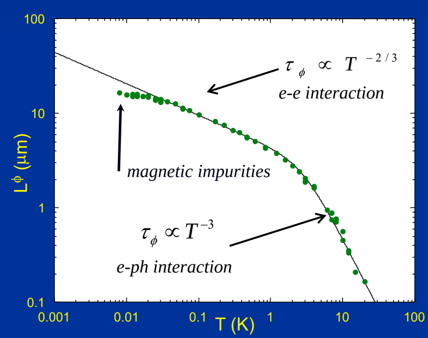
Average on the trajectories and on the dynamics of external degrees of freedom

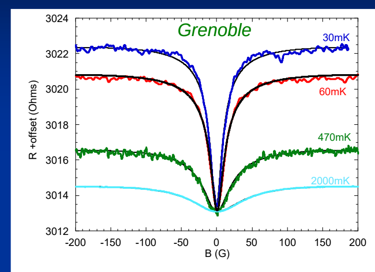
$$\left\langle e^{i\Delta\varphi(t)} \right\rangle \sim e^{-\frac{1}{2}\left\langle \Delta\varphi^2(t) \right\rangle} \sim e^{-t/\tau_\phi}$$

Magnetotransport gives access to the phase coherence length

$$\delta G(B) = f_{2d}\left(\frac{BL_\phi^2}{\phi_0}\right) \longrightarrow L_\phi(T)$$

$$\delta G(B) = f_{1d}\left(\frac{BWL_\phi}{\phi_0}\right)$$

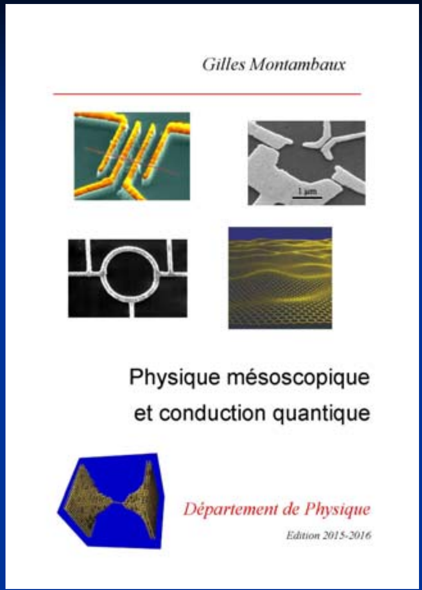




quasi-1D wires

e-e e-phonon

$$\boxed{\frac{1}{\tau_\phi(T)} = AT^{2/3} + BT^3}$$



https://gilles.montambaux.com/enseignement/physique-mesoscopique/