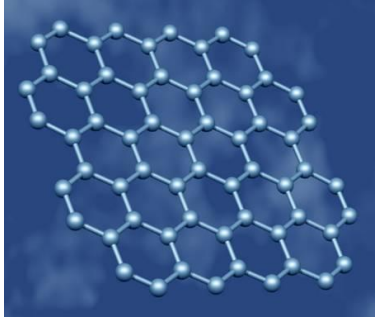


La révolution du graphène II

Gilles Montambaux, Laboratoire de Physique des Solides, Orsay, France

UNIVERSITÉ PARIS SUD

cnrs

users.lps.u-psud.fr/montambaux

aujourd'hui...

Propriétés de transport

Effet tunnel de Klein

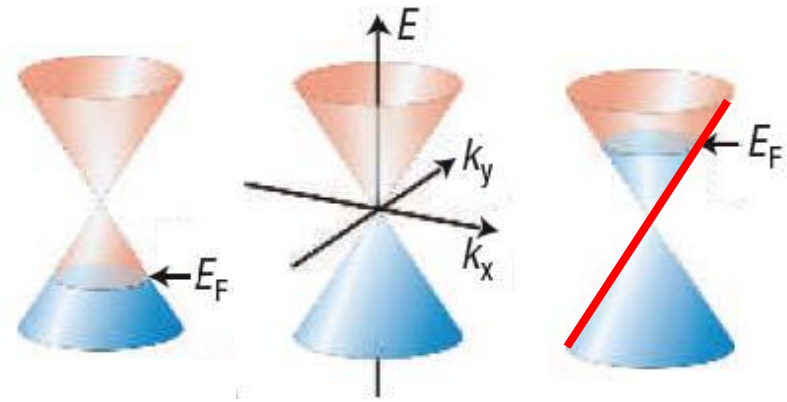
Rubans de graphène – états de bords

Propriétés de transport

Rappels

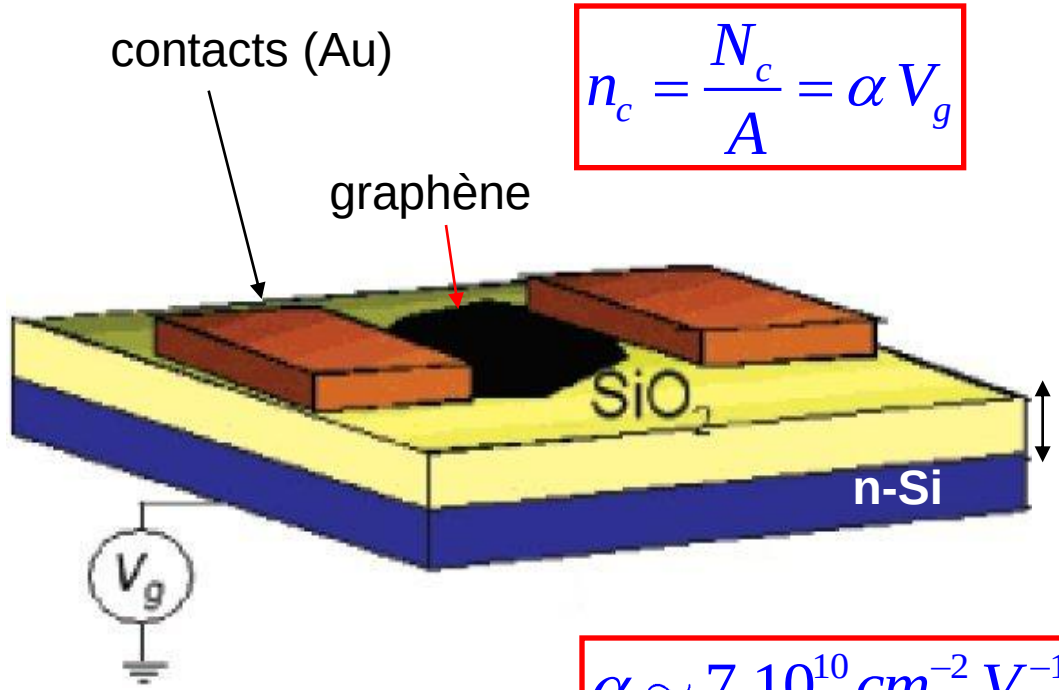
$$\varepsilon = \pm \hbar c |q|$$

$$v_F = c = \frac{3}{2}\frac{ta}{\hbar}$$



$$\varepsilon_F = \pm \hbar v_F \sqrt{\pi n_c}$$

$$n_c = \frac{N_c}{A} = \alpha V_g$$

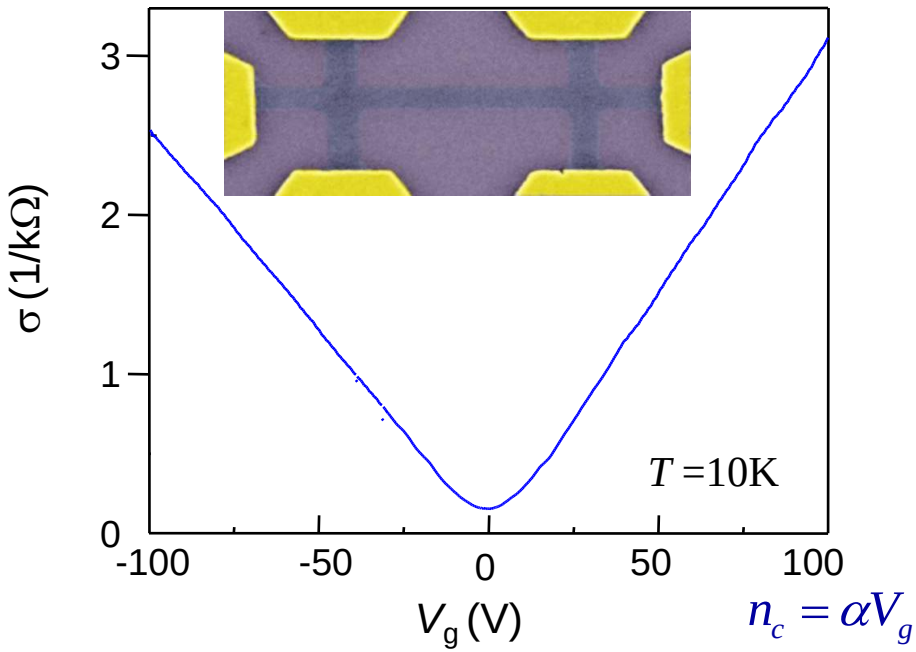


$$\alpha \sim 7.10^{10} cm^{-2}.V^{-1}$$

Conductivity

$$\varepsilon_F = \hbar v_F \sqrt{\pi n_c}$$

$$n_c = \alpha V_g$$

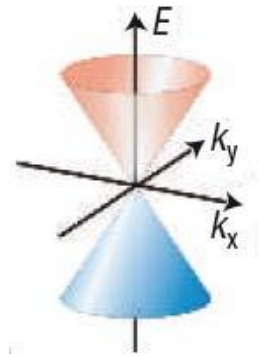


$$\sigma(\varepsilon_F) = 2\frac{e^2}{h} k_F l_e$$

$l_e(\varepsilon_F)$???

$\sigma(T)$???

Physics of the neutrality point



$E_F=0$

Thermally activated carriers

$$n_e(T) = n_h(T) = \frac{2\pi}{27}\left(\frac{k_B T}{ta}\right)^2$$

Specific heat

$$C(T) = \frac{8\xi(3)}{\pi} k_B \left(\frac{k_B T}{ta}\right)^2$$

Conductivity

$$\sigma(\varepsilon_F = 0, T) = 4\ln 2 \frac{e^2}{h}\frac{l_e}{\hbar v_F} k_B T$$

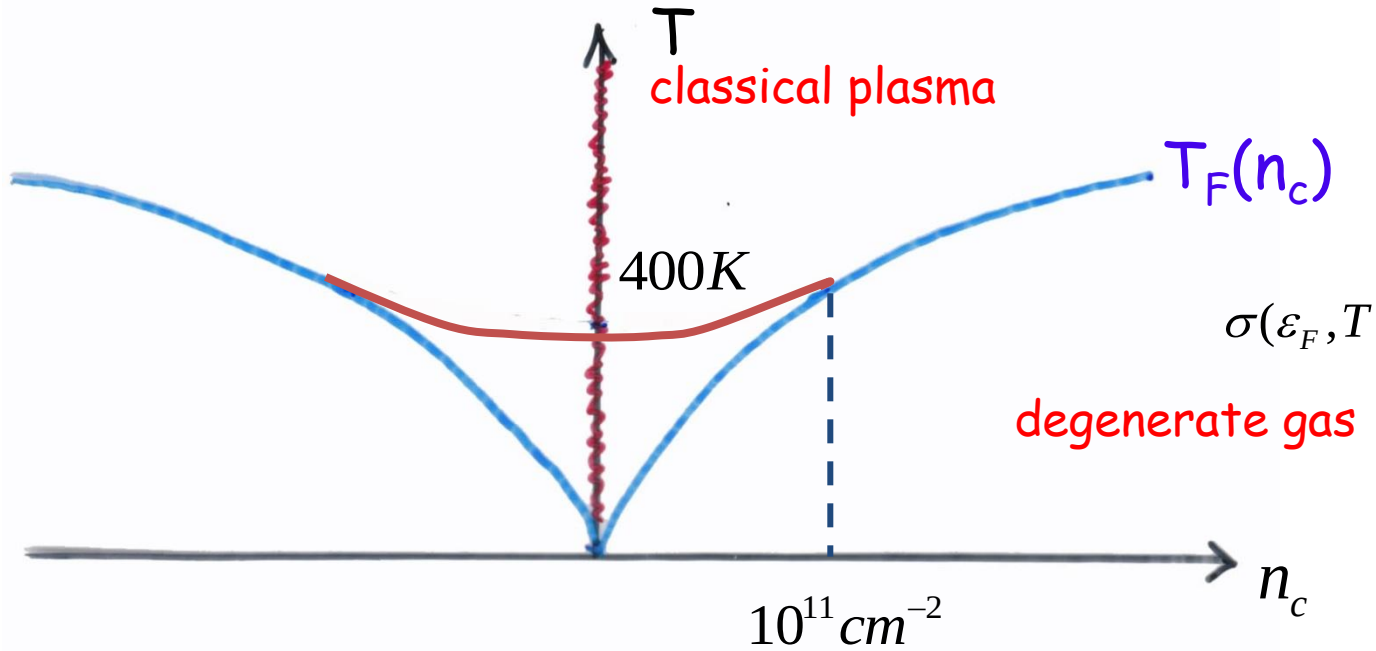
$$\sigma(\varepsilon_F, T = 0) = 2\frac{e^2}{h} k_F l_e$$

$$\sigma(T) = -\int \sigma(\varepsilon)\frac{\partial f}{\partial \varepsilon} d\varepsilon$$

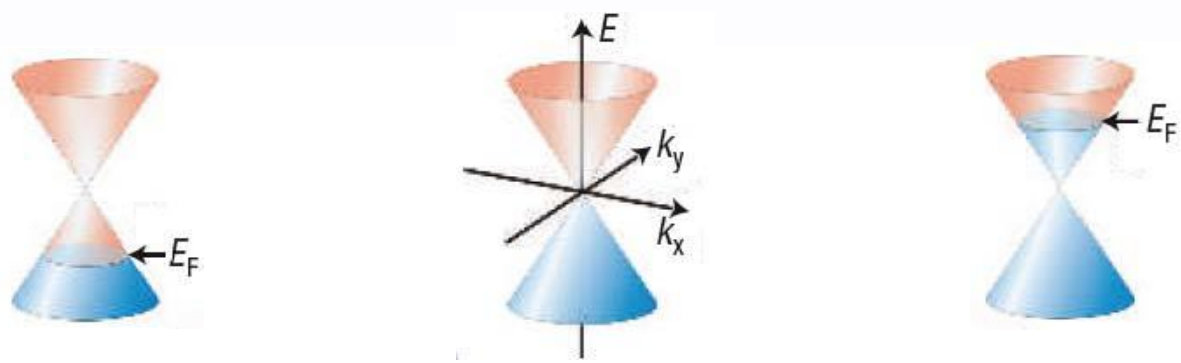
$$\sigma(\varepsilon_F = 0, T) = 4\ln 2 \frac{e^2}{h} \frac{l_e}{\hbar v_F} k_B T$$

$$\varepsilon_F = \hbar v_F \sqrt{\pi n_c}$$

$n_c = \alpha V_g$



$$\sigma(\varepsilon_F, T = 0) = 2\frac{e^2}{h} k_F l_e$$



$T > T_F$

classical plasma

$T < T_F$

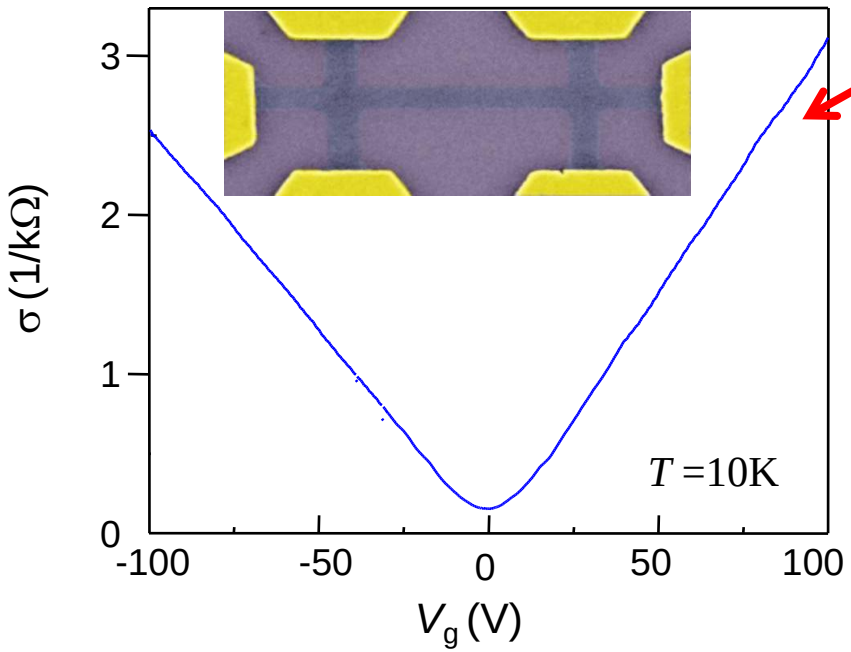
degenerate gas

Conductivity

$$\varepsilon_F = \hbar v_F \sqrt{\pi n_c}$$

$$n_c = \alpha V_g$$

$\sigma = n(V_g) e\mu$



$$\sigma(\varepsilon_F) = 2\frac{e^2}{h} k_F l_e$$

$l_e(\varepsilon_F)$???

Very small temperature dependence

$l_e \sim Cte$ $\quad \sigma(\varepsilon_F) \propto \varepsilon_F \propto \sqrt{V_g}$

$l_e \propto \frac{1}{k_F}$ $\quad \sigma(\varepsilon_F) \propto Cte$ neutral impurities

$l_e \propto k_F$ $\quad \sigma(\varepsilon_F) \propto \varepsilon_F^2 \propto V_g$ charged screened impurities

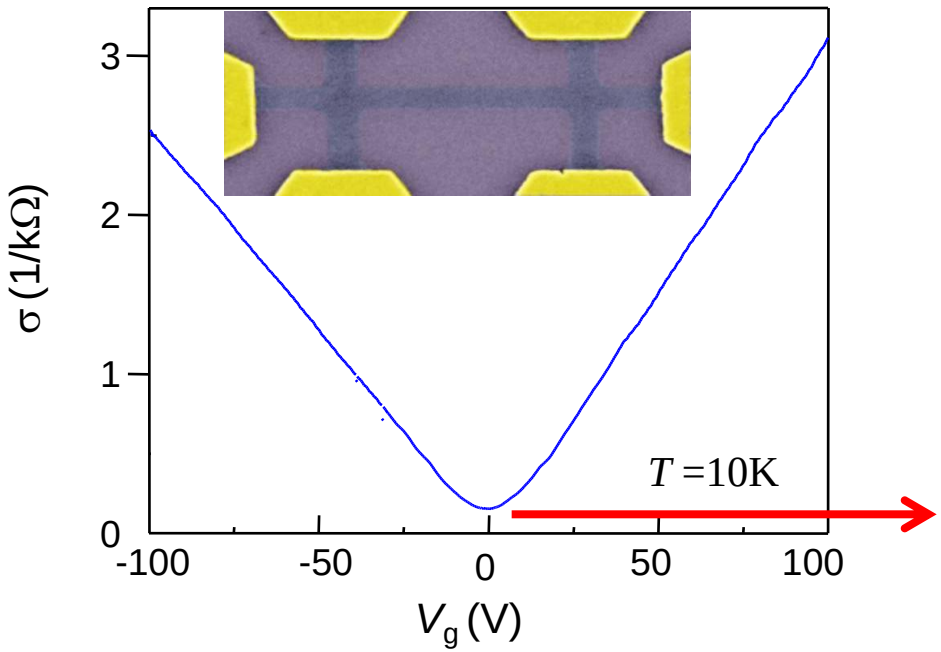
Nomura, MacDonald, PRL 2007

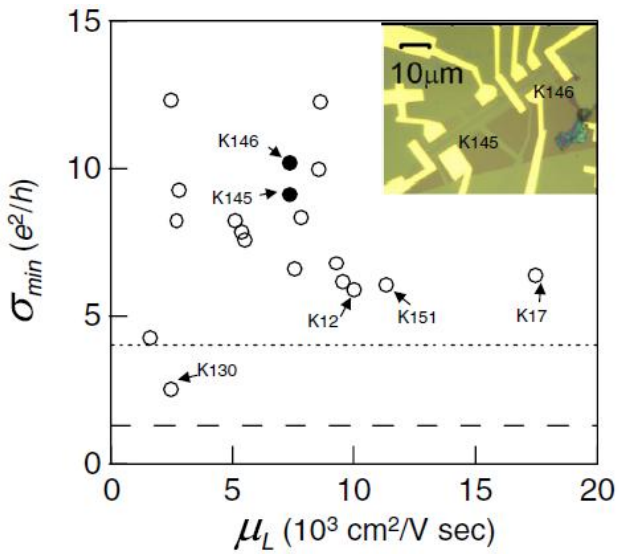
should depend on dielectric constant…

see also M.Monteverde et al. PRL (2010)
resonant impurities

Minimum de conductivité

devrait être nulle au point de neutralité ?

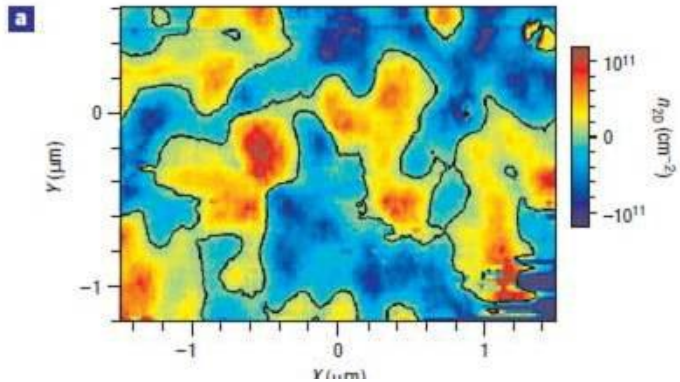


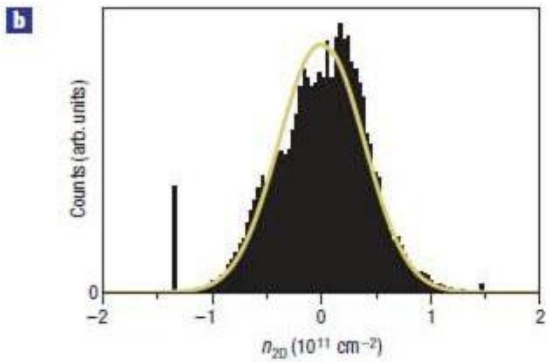


Tan et al. PRL 99, 246803 (2007)

Explication : « flaques » de trous et d'électrons

$n_c = 0$ $\delta n_c \neq 0$

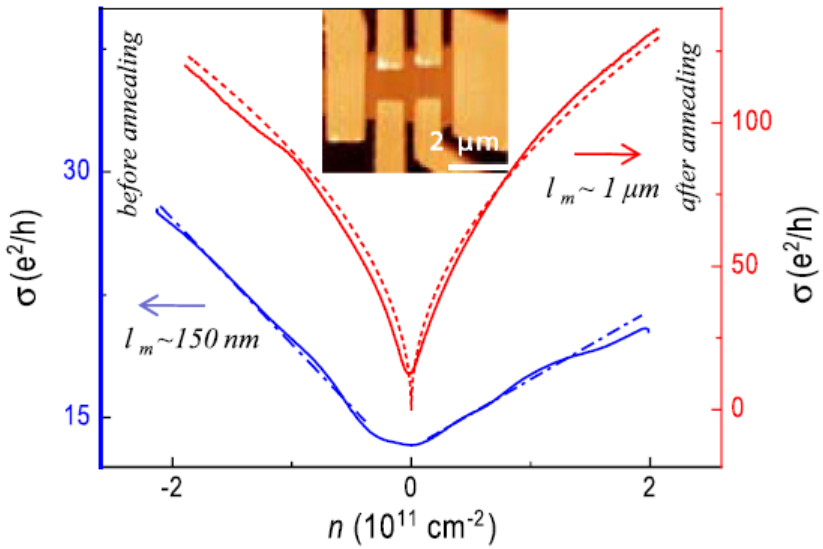




Yacoby et al., Nature Phys. 2008

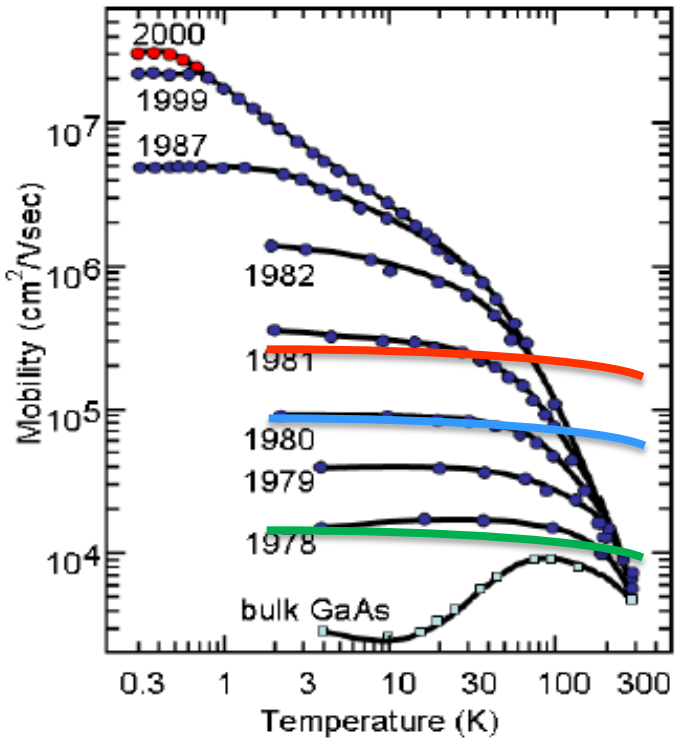
Suspended graphene

Bolotin et al., Columbia PRL 101, 096802 (2008)



$$\sigma = 2\frac{e^2}{h} k_F \, l_e(k_F)$$ diffusive

$$\sigma = \frac{4}{\pi}\frac{e^2}{h} k_F L$$ ballistic



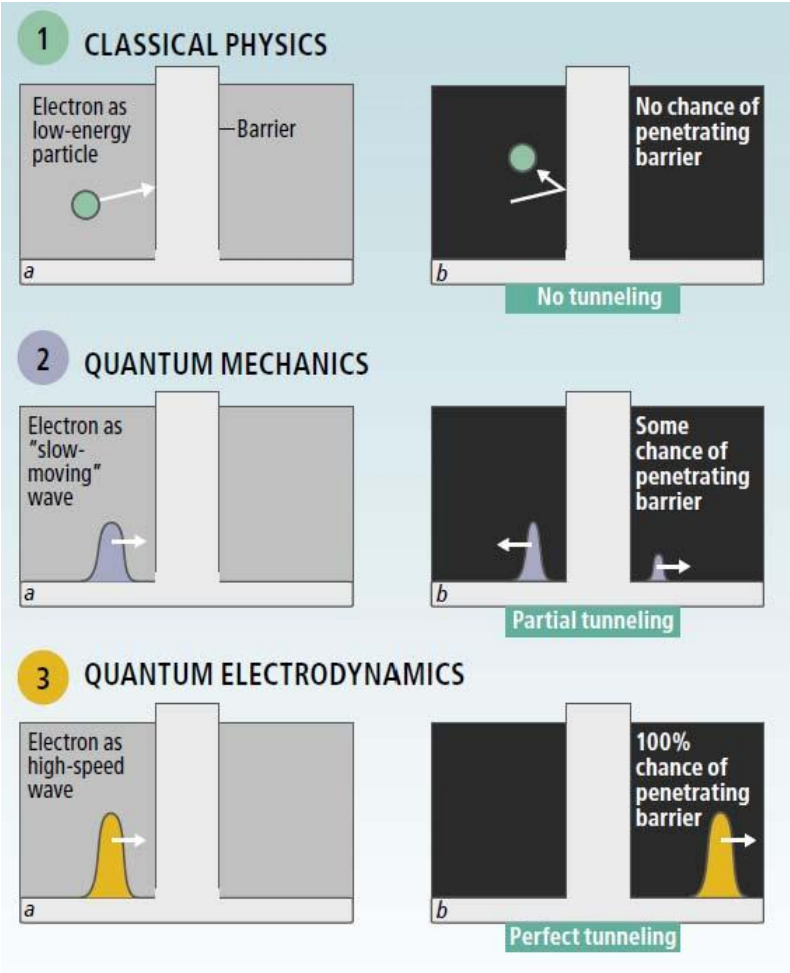
GaAs mobility, Pfeiffer et al.

Suspended graphene (2008)

Graphene on h-BN (2010)

Graphene on SiO2 (2004)

Klein tunneling effect



Oskar Klein (1894-1977)

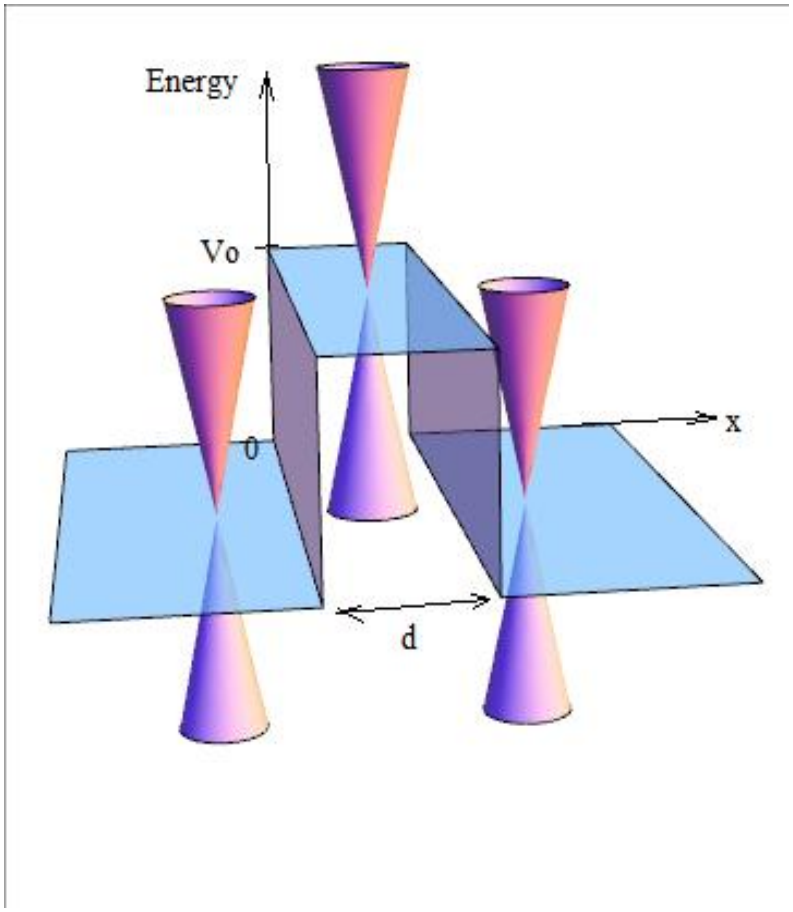
Die Reflexion von Elektronen an einem Potentialsprung nach der relativistischen Dynamik von Dirac.

Von **O. Klein** in Kopenhagen.

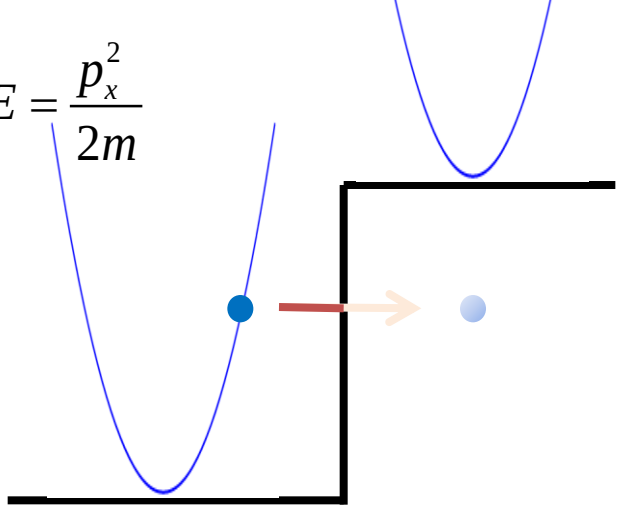
(Eingegangen am 24. Dezember 1928.)

1) Potential step

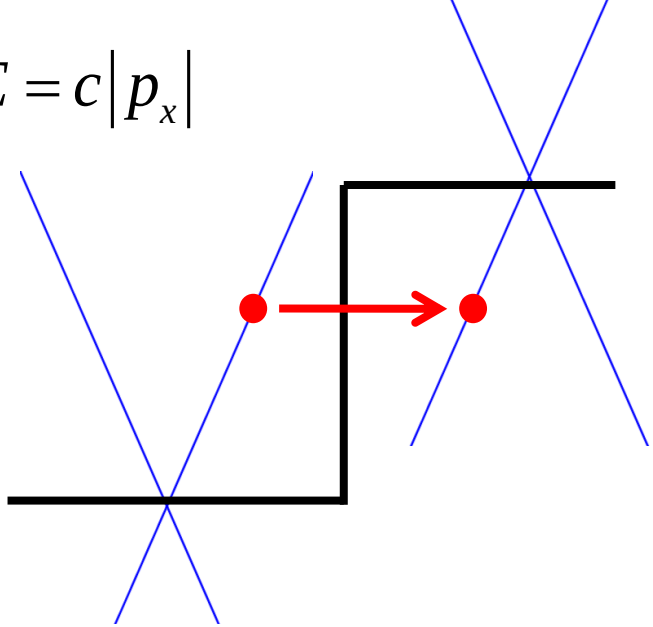
2) Potential barrier



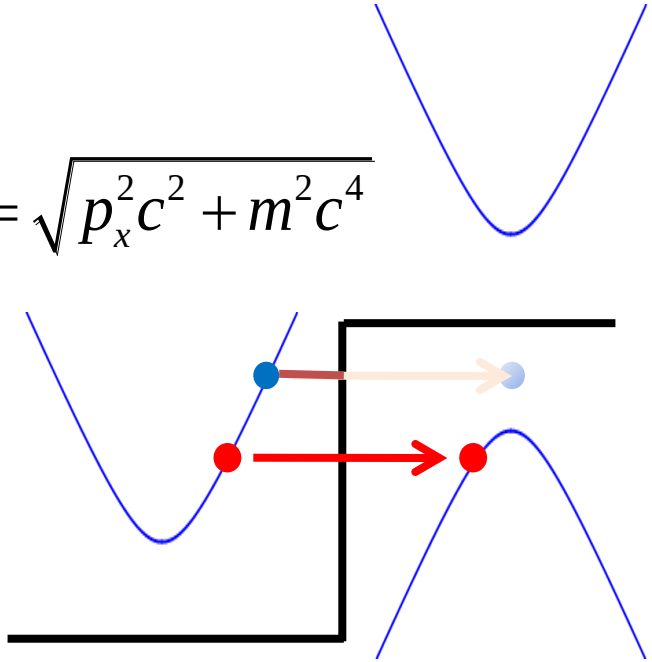
$$E = \frac{p_x^2}{2m}$$



$$E = c\left|p_x\right|$$



$$E = \sqrt{p_x^2 c^2 + m^2 c^4}$$

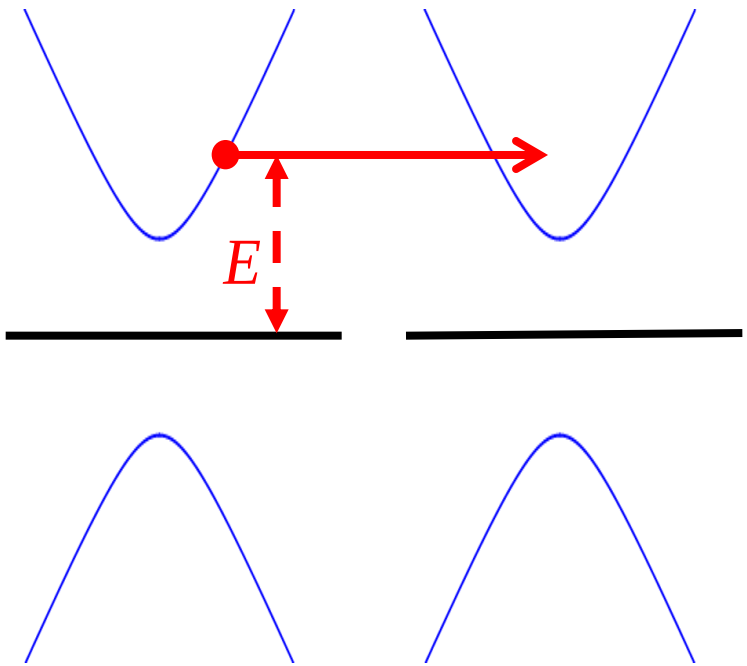


Evanescent mode

Transmission

$$E = \sqrt{p_x^2 c^2 + m^2 c^4}$$

$$E > V_0 + mc^2$$



$$E = \sqrt{p_x^2 c^2 + m^2 c^4}$$

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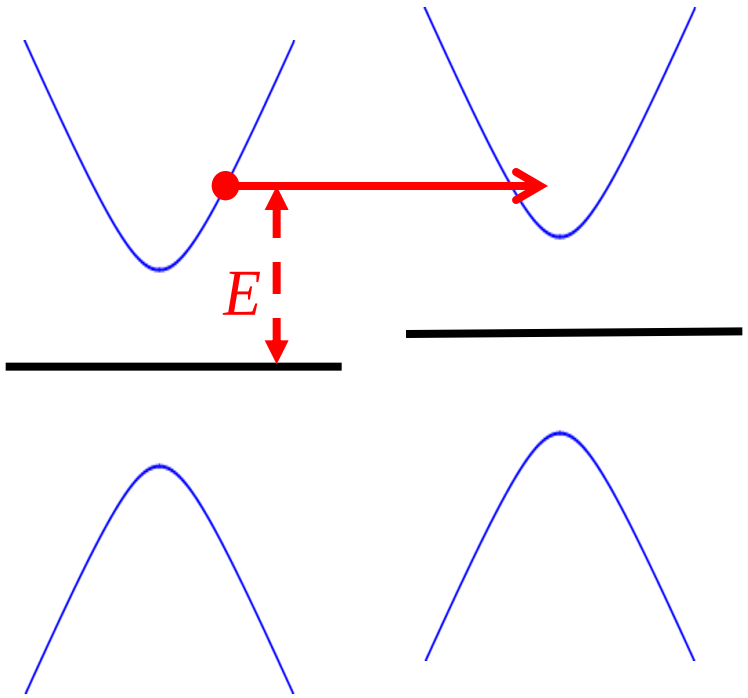

$$E = \sqrt{p_x^2 c^2 + m^2 c^4}$$

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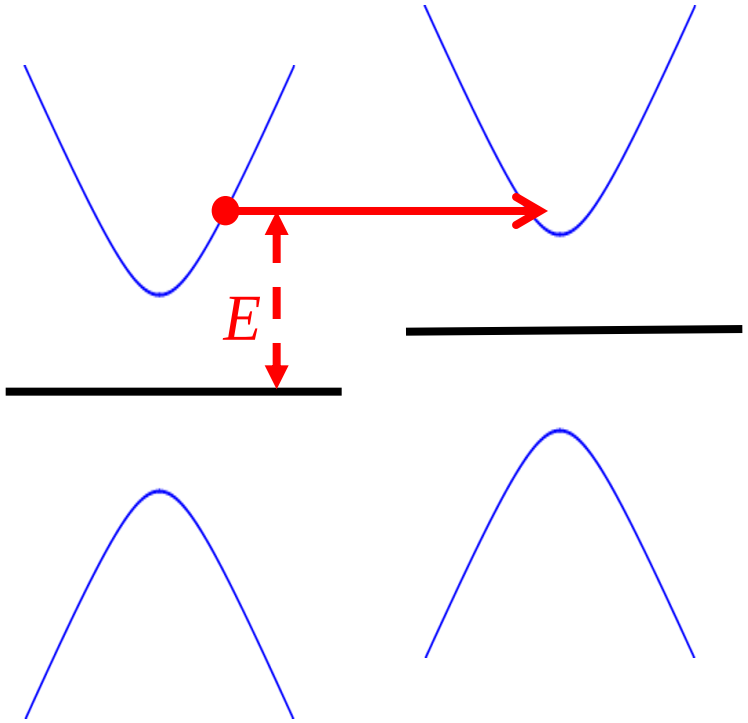
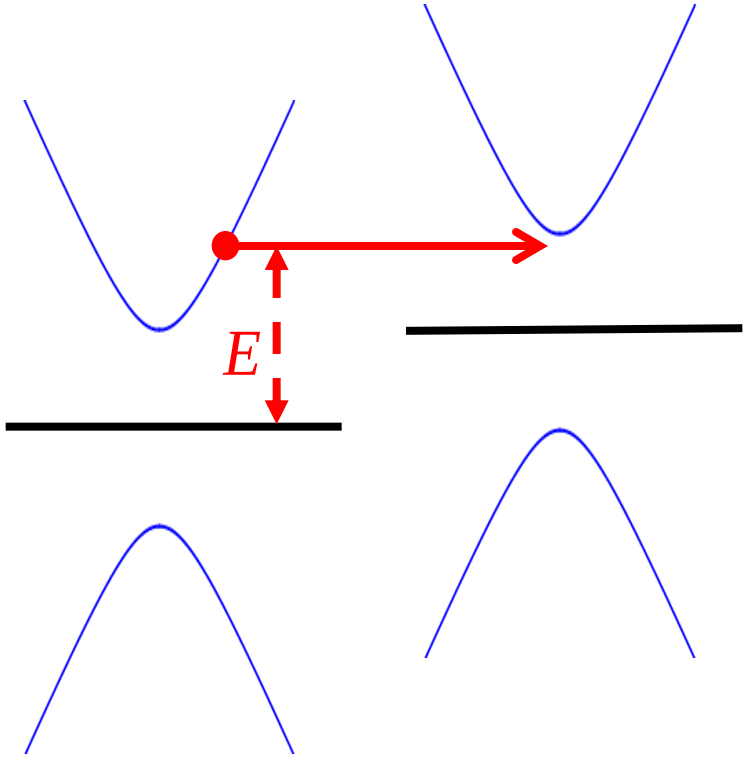
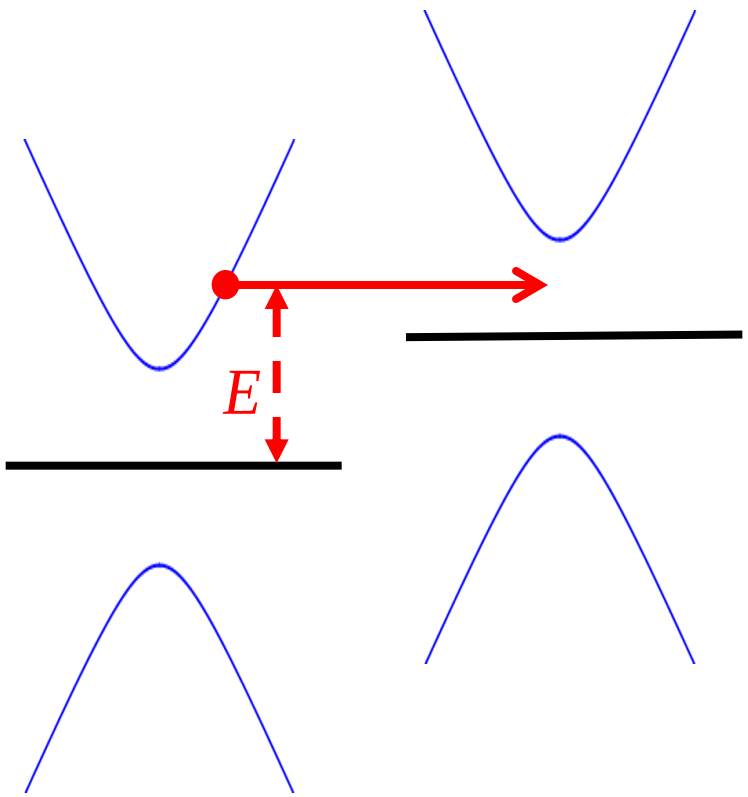


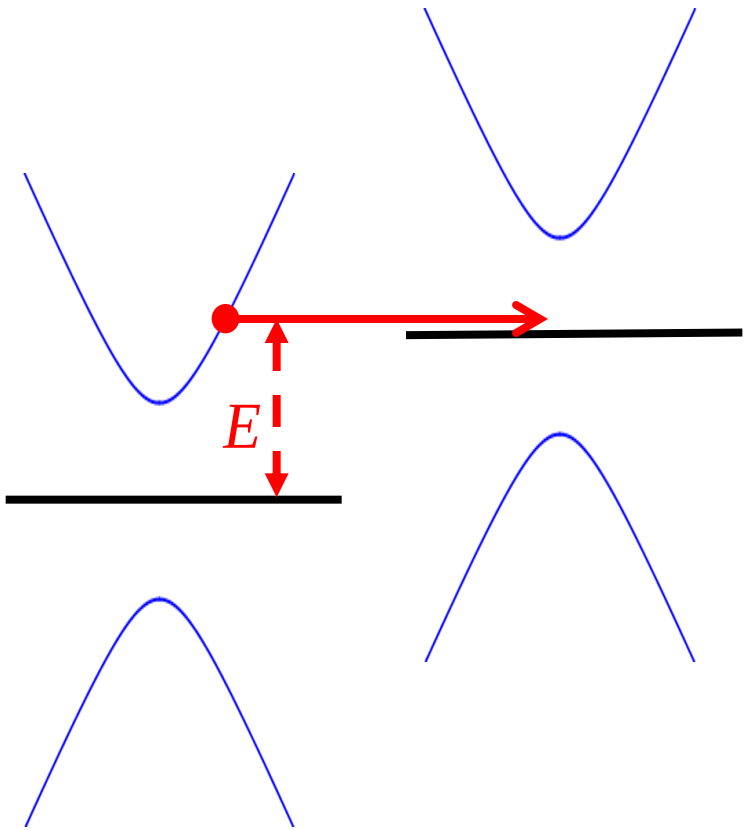
$$E = \sqrt{p_x^2 c^2 + m^2 c^4}$$



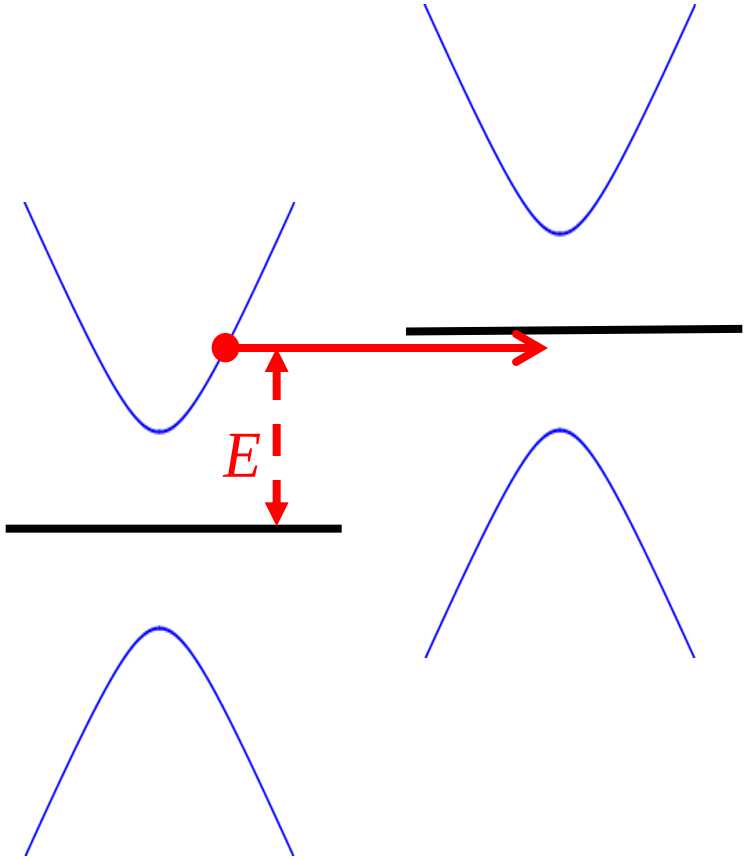
$$E = \sqrt{p_x^2 c^2 + m^2 c^4}$$



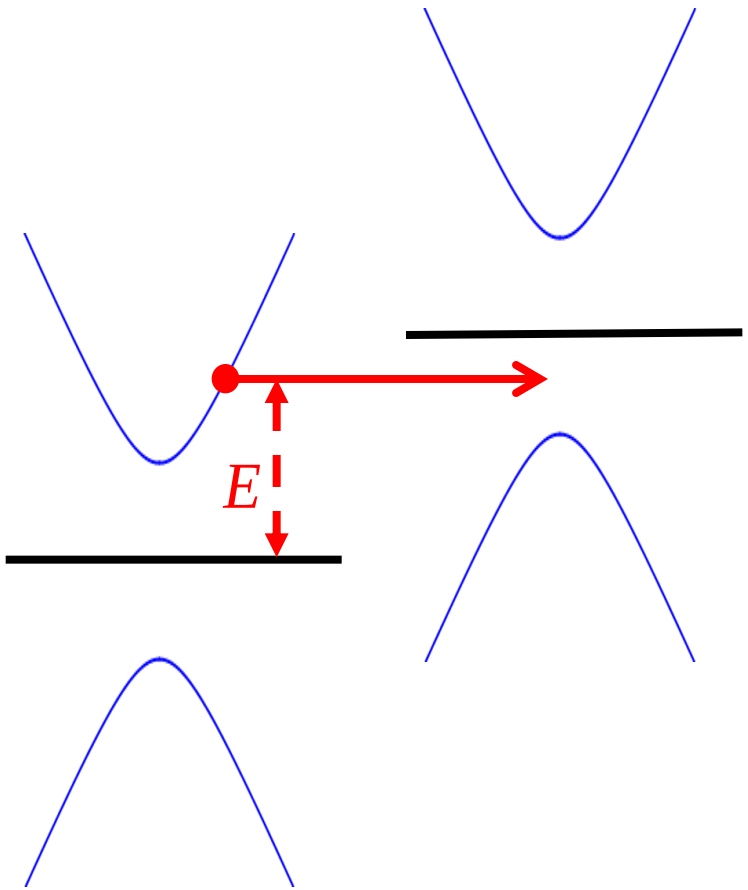
$$E = \sqrt{p_x^2 c^2 + m^2 c^4}$$



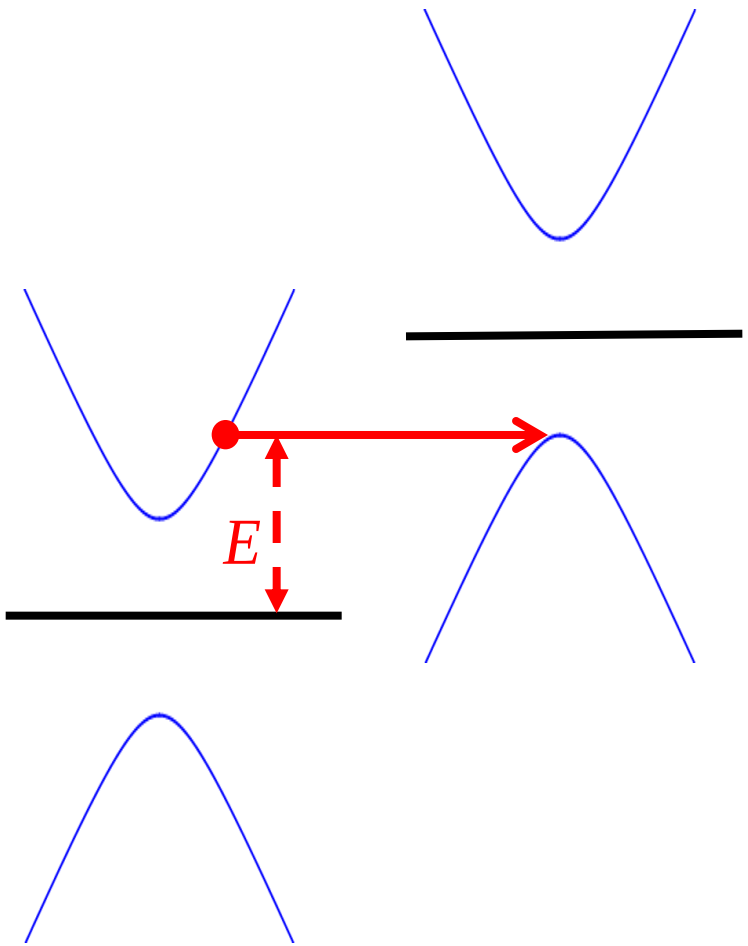
$$E = \sqrt{p_x^2 c^2 + m^2 c^4}$$



$$E = \sqrt{p_x^2 c^2 + m^2 c^4}$$

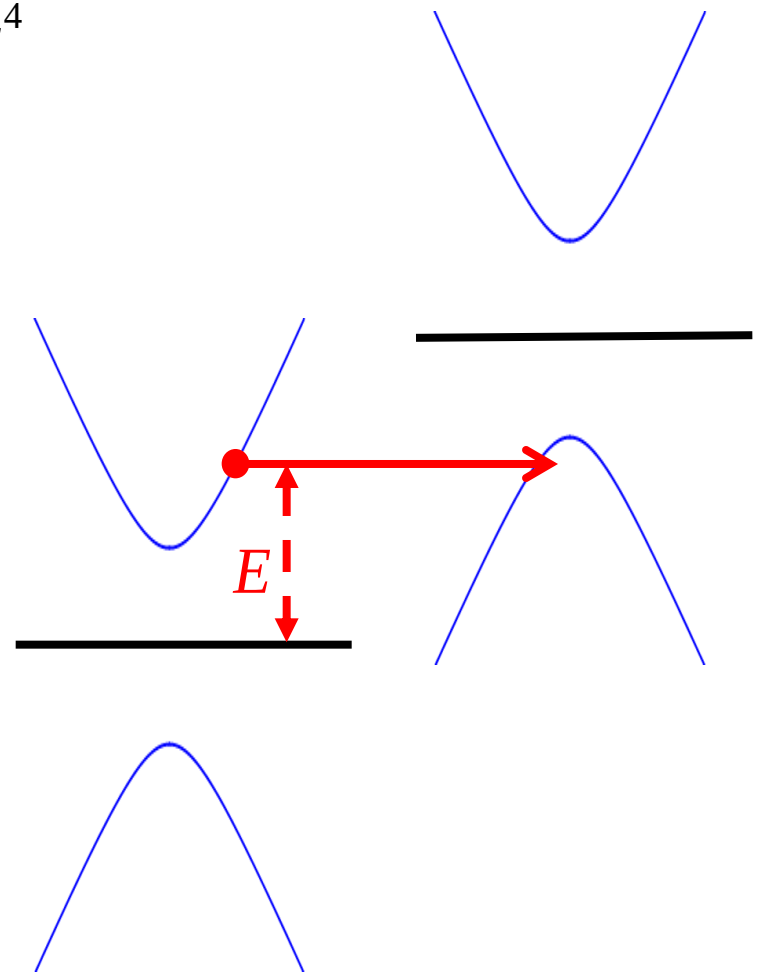


$$E = \sqrt{p_x^2 c^2 + m^2 c^4}$$



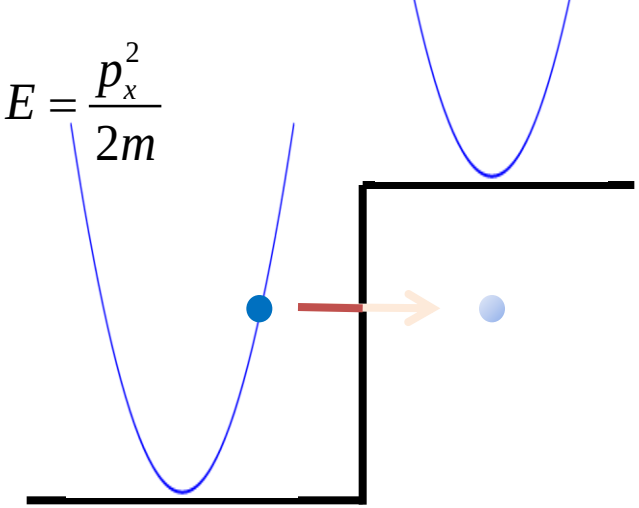
$$V_0 = E + mc^2$$

$$E = \sqrt{p_x^2 c^2 + m^2 c^4}$$

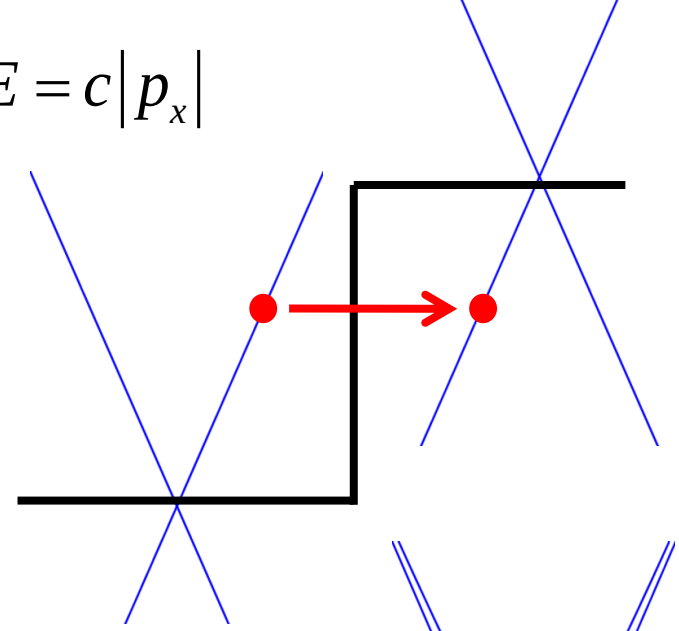


$$V_0 > E + mc^2$$

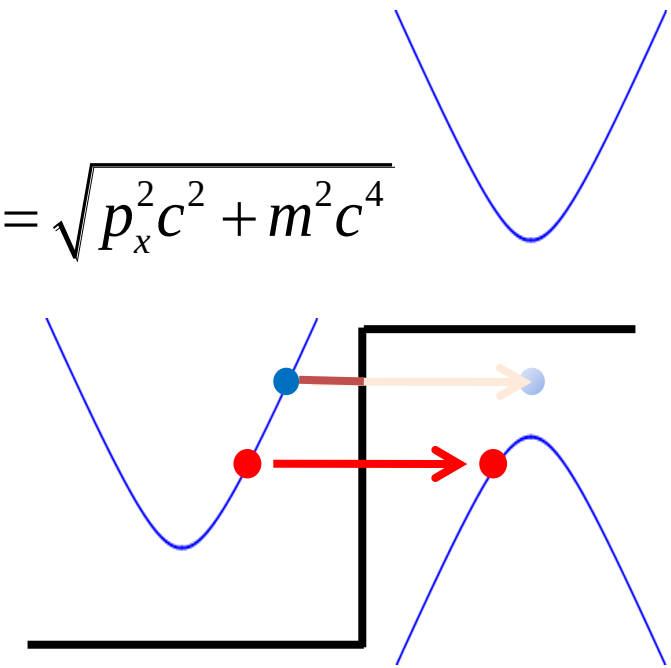
$$E = \frac{p_x^2}{2m}$$



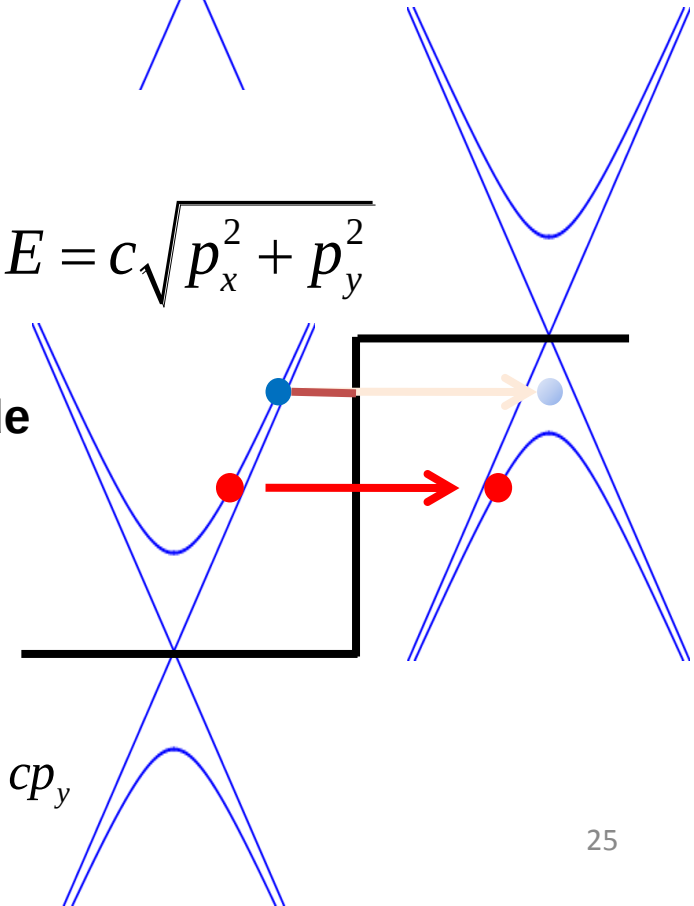
$$E = c\left|p_x\right|$$



$$E = \sqrt{p_x^2 c^2 + m^2 c^4}$$



$$E = c\sqrt{p_x^2 + p_y^2}$$



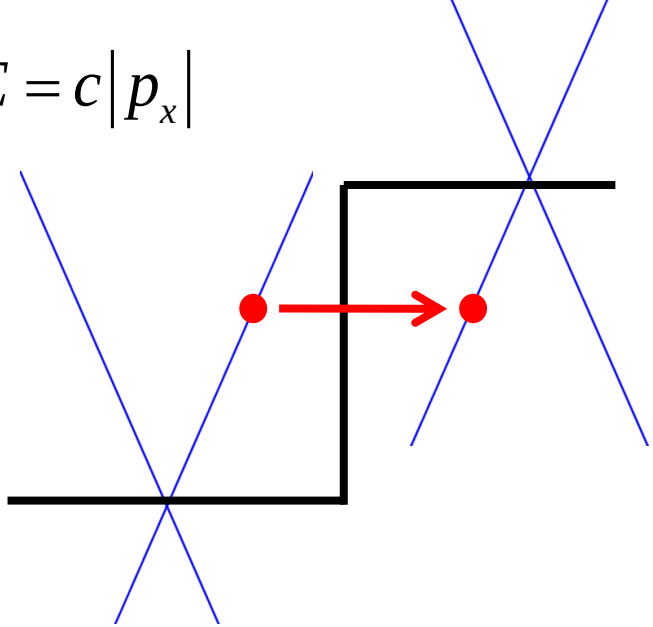
Evanescent mode

Transmission

$$\left|E - V_0\right| > mc^2 \qquad \left|E - V_0\right| > cp_y$$

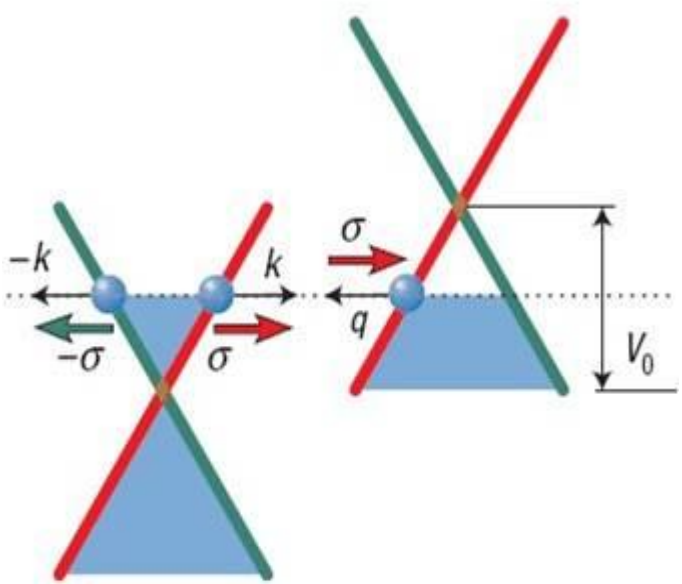
Velocity

$$E = c\left|p_x\right|$$



$$E = \alpha\hbar v_F \left|\vec{k}\right|$$

$$\vec{v} = \frac{1}{\hbar}\nabla_{\vec{k}} E$$



Fonctions d'onde

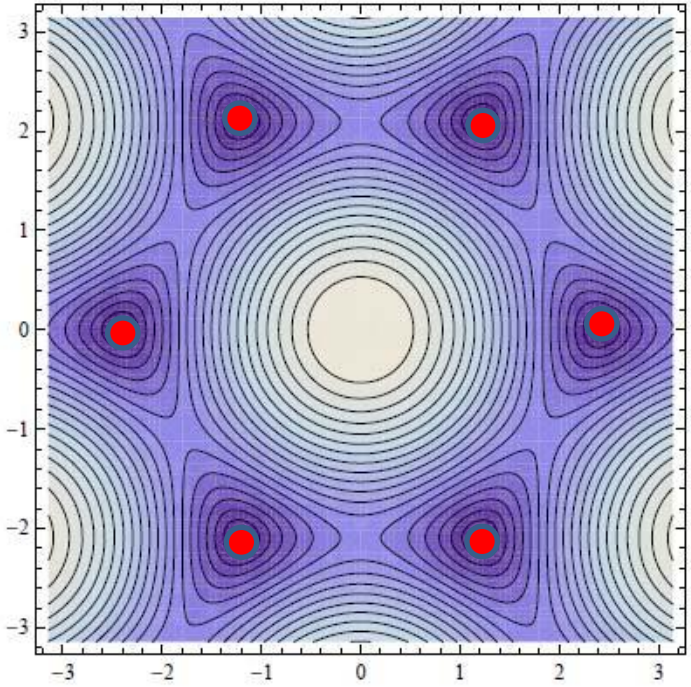
$$H_k = \begin{pmatrix} 0 & f(\vec{k}) \\ f^*(\vec{k}) & 0 \end{pmatrix}$$

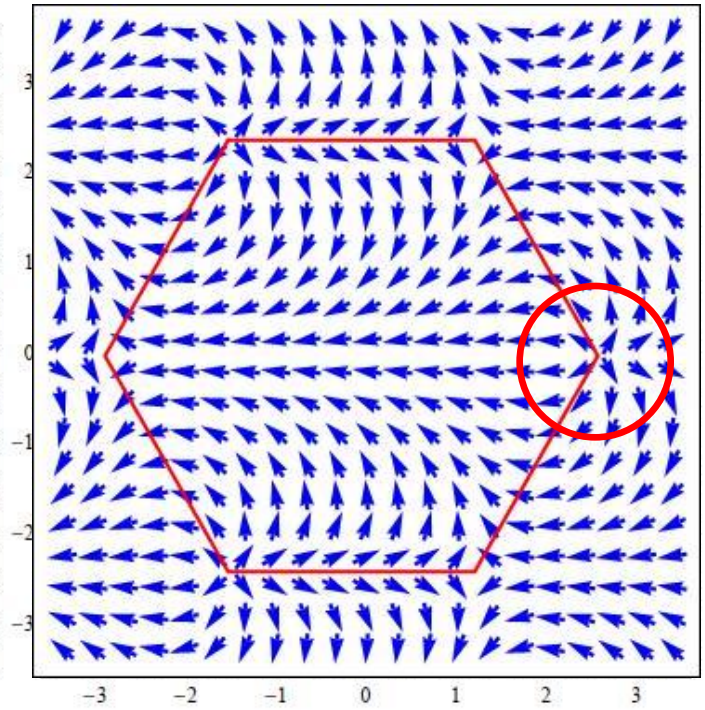
$$\psi_{\vec{k}}(\vec{r}) = \frac{e^{i\vec{k}.\vec{r}}}{\sqrt{2}} \begin{pmatrix} 1 \\ \alpha e^{i\vartheta_{\vec{k}}} \end{pmatrix}$$

$$H_k = \left|\varepsilon(\vec{k})\right| \begin{pmatrix} 0 & e^{-i\vartheta_{\vec{k}}} \\ e^{i\vartheta_{\vec{k}}} & 0 \end{pmatrix}$$

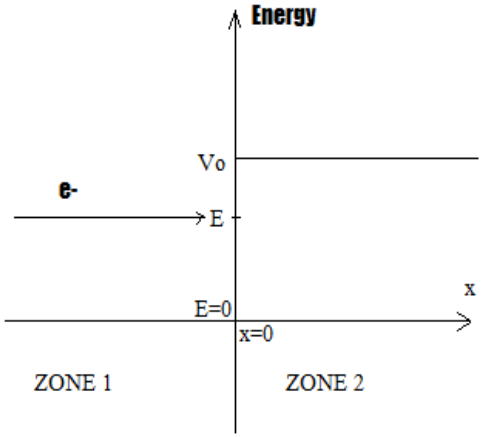
$$\tan\theta_{\vec{k}} = \frac{k_y}{k_x}$$

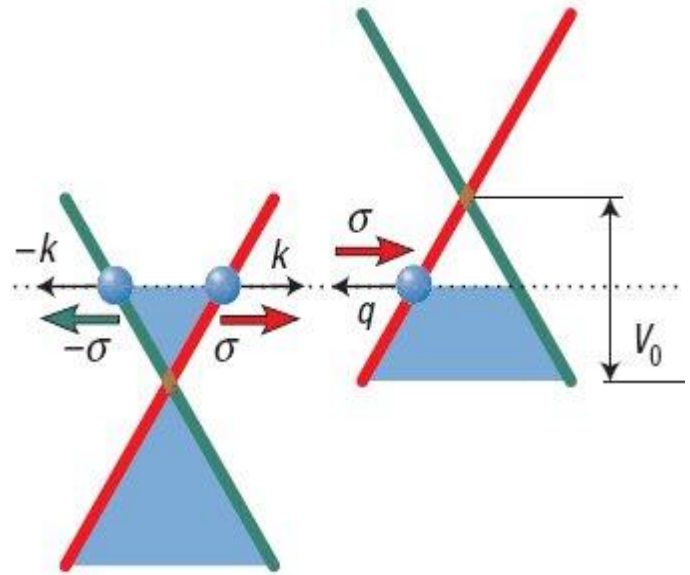
$$\theta_{\vec{k}} = \arg[f(\vec{k})]$$

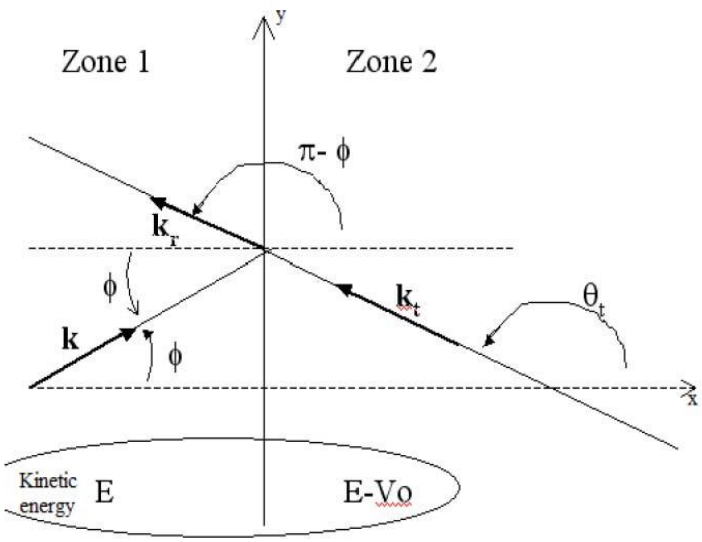




Potential step







$$\psi_I(\boldsymbol{r}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ se^{i\phi} \end{pmatrix} e^{i(k_x x + k_y y)} + \frac{r}{\sqrt{2}}\begin{pmatrix} 1 \\ se^{i(\pi-\phi)} \end{pmatrix} e^{i(-k_x x + k_y y)}$$

$$\psi_{II}(\boldsymbol{r}) = \frac{a}{\sqrt{2}}\begin{pmatrix} 1 \\ s'e^{i\theta} \end{pmatrix} e^{i(q_x x + k_y y)}$$

$$T(\phi) = -\frac{\cos\phi\cos\theta_t}{\sin^2\left(\frac{\phi+\theta_t}{2}\right)}$$

$$E\sin\phi = (V_0 - E)\sin\theta_t$$

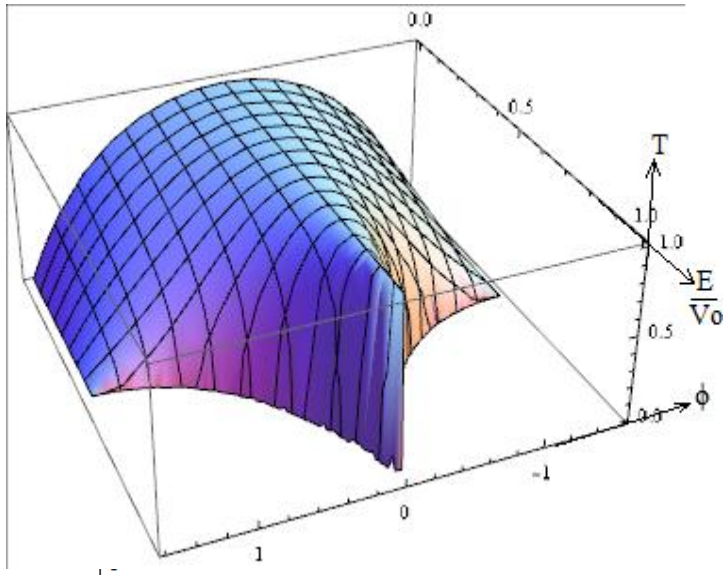
+ Continuity equations

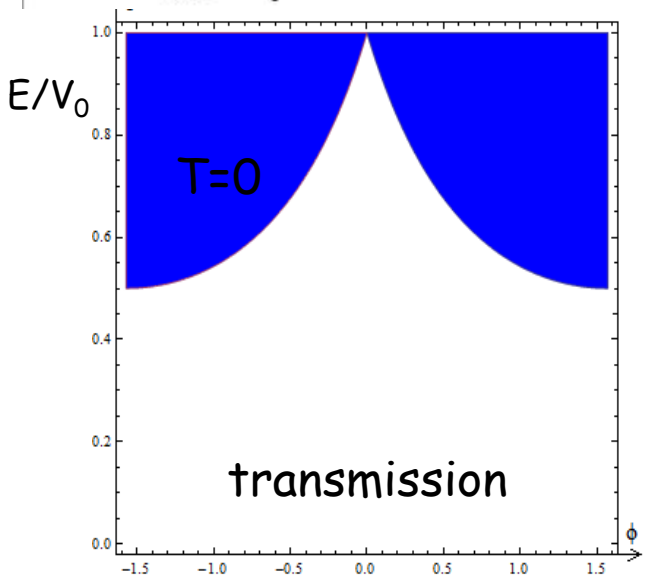
Potential step

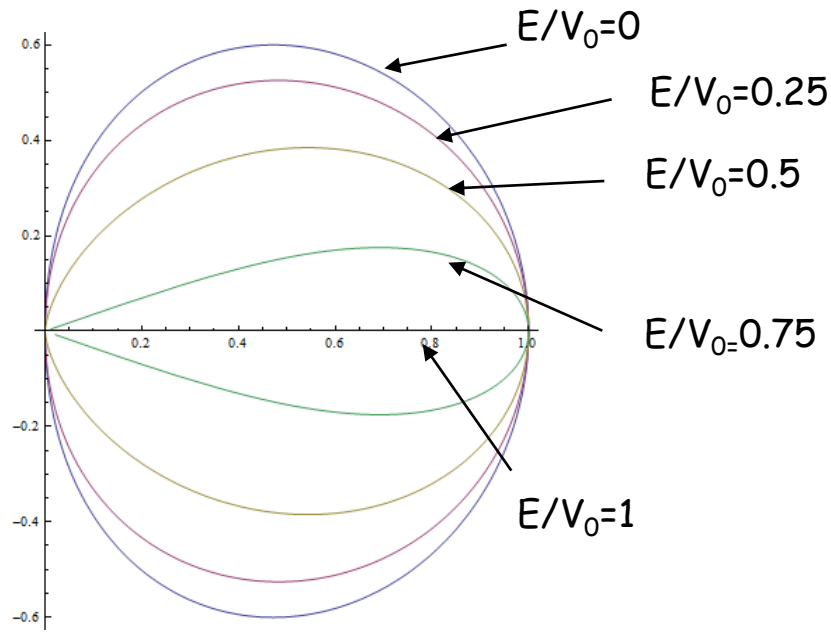
$T(\phi)$

$$T(\phi) = -\frac{\cos\phi\cos\theta_t}{\sin^2\left(\frac{\phi+\theta_t}{2}\right)}$$

$$E\sin\phi = (V_0 - E)\sin\theta_t$$



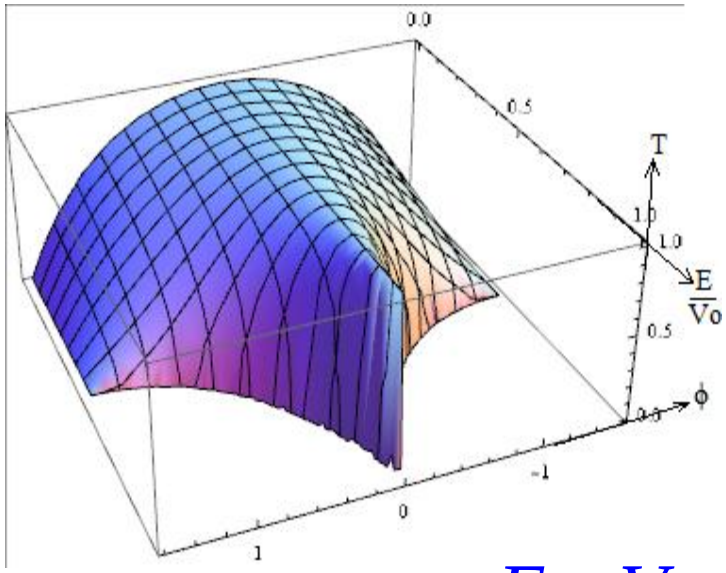




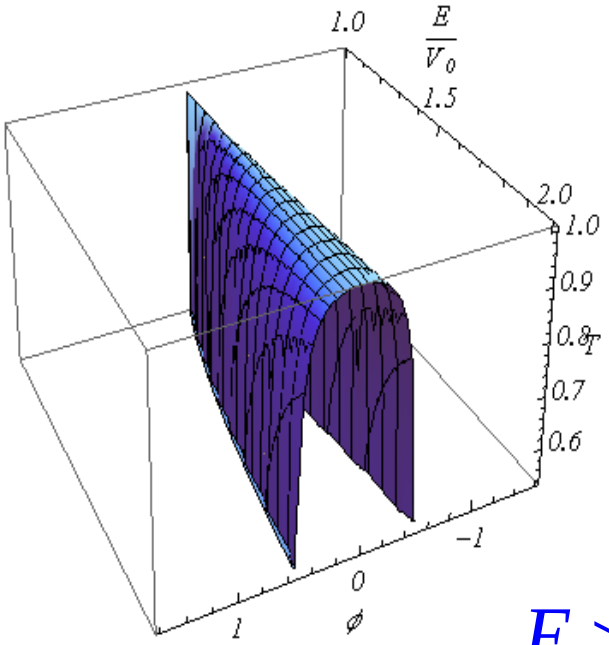
When E approches V_0, more and more evanescent modes

Potential step

T(ϕ)



$E < V_0$



$E > V_0$

$$T(\phi) = -\frac{\cos\phi\cos\theta_t}{\sin^2\left(\frac{\phi+\theta_t}{2}\right)}$$

$$E\sin\phi = (V_0 - E)\sin\theta_t$$

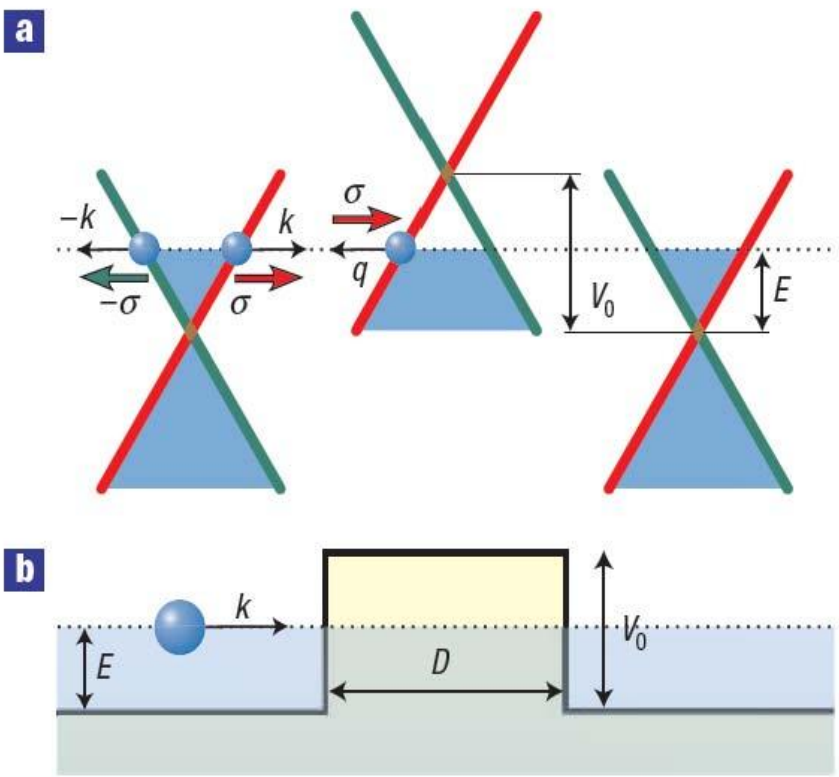
$$T(\phi) = \frac{\cos\phi\cos\theta_t}{\cos^2\left(\frac{\phi+\theta_t}{2}\right)}$$

$$E\sin\phi = (E - V_0)\sin\theta_t$$

Potential barrier

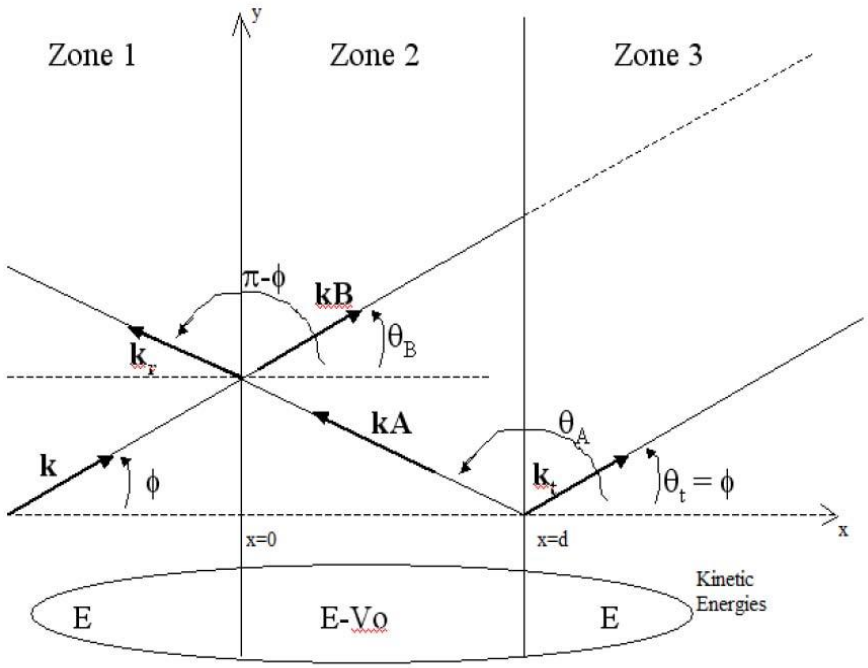
$$\psi_I(\boldsymbol{r}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ se^{i\phi} \end{pmatrix} e^{i(k_x x + k_y y)} + \frac{r}{\sqrt{2}}\begin{pmatrix} 1 \\ se^{i(\pi-\phi)} \end{pmatrix} e^{i(-k_x x + k_y y)}$$

$$\psi_{\mathrm{II}}(\boldsymbol{r}) = \frac{a}{\sqrt{2}}\begin{pmatrix} 1 \\ s'e^{i\theta} \end{pmatrix} e^{i(q_x x + k_y y)} + \frac{b}{\sqrt{2}}\begin{pmatrix} 1 \\ s'e^{i(\pi-\theta)} \end{pmatrix} e^{i(-q_x x + k_y y)}$$



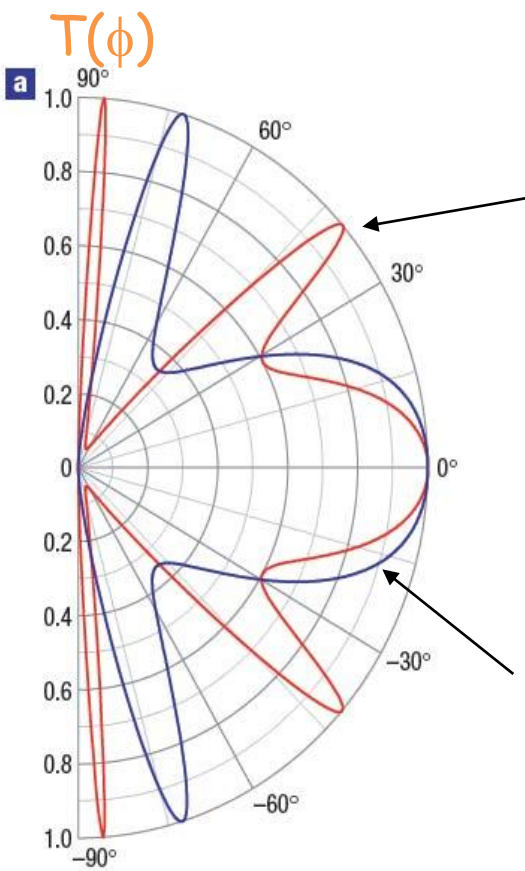
$$\psi_{\mathrm{III}}(\boldsymbol{r}) = \frac{t}{\sqrt{2}}\begin{pmatrix} 1 \\ se^{i\phi} \end{pmatrix} e^{i(k_x x + k_y y)}$$

+ Continuity equations
→



J.N. Fuchs et P. Allain, Klein Tunneling in graphene : optics with massless electrons EPJB, arXiv:1104.5632

Potential barrier



E/V_0=0.42, $l=L/(hv_F/V_0)$=4.8

E/V_0=0.29, l=6.91

Katsnelson, Novoselov and Geim, Nat. Phys. 2, 620 (2006)

$$E\sin\phi = (V_0 - E)\sin\theta_t$$

$$q_x = (V_0 - E)\cos\theta_t$$

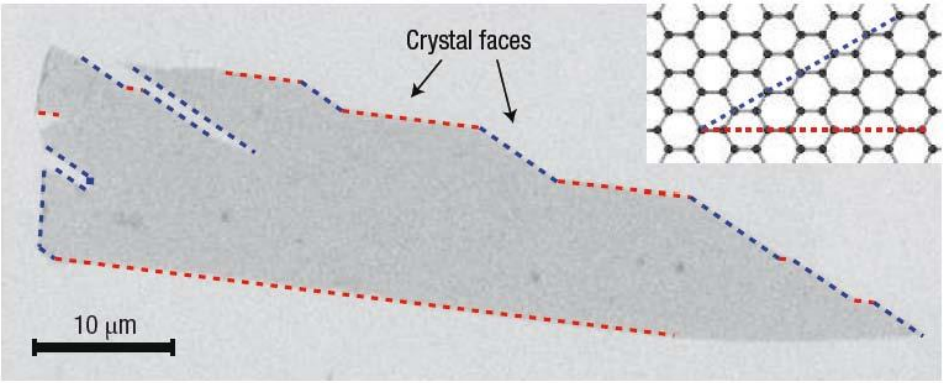
$$T(\phi) = \frac{\cos^2\phi\cos^2\theta_t}{\cos^2 q_x L\cos^2\phi\cos^2\theta_t + \sin^2 q_x L\,(1+\sin\phi\sin\theta_t)^2}$$

Les rubans de graphène

Zig-zag

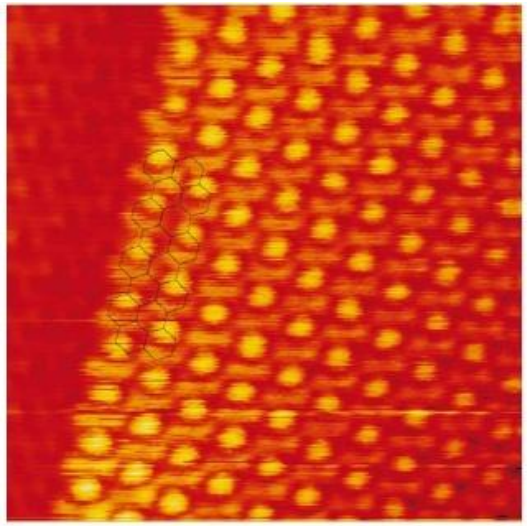
Mosaique romaine, El Djem, Tunisie

Graphene ribbons

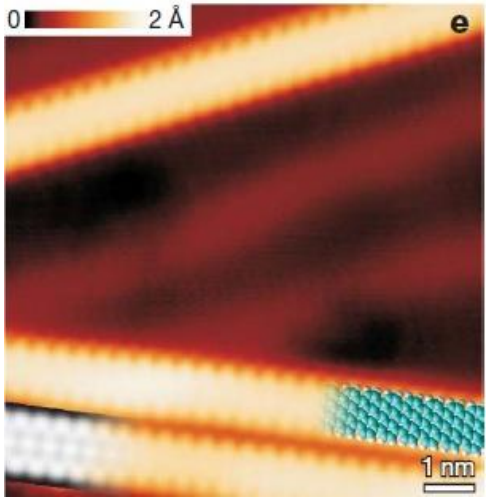


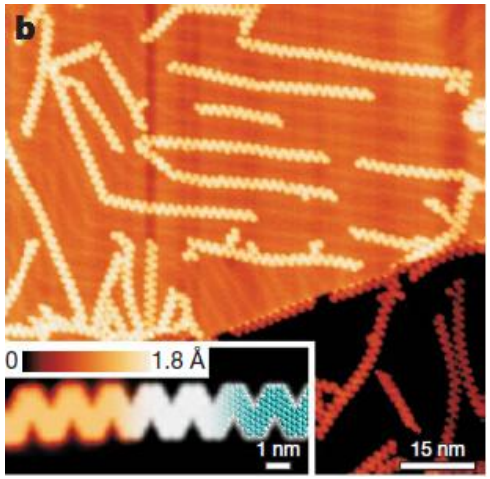
From Geim, Novoselov, Nature mat. 6, 183 (2007)

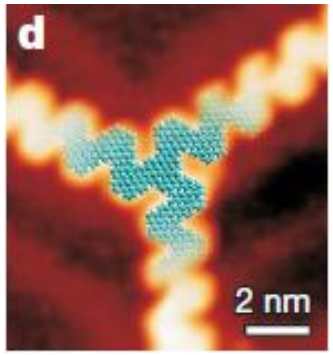
STM image, 5.6 × 5.6 nm^2



Y. Kobasashi *et al.* PRB **71**, 193406 (2005).



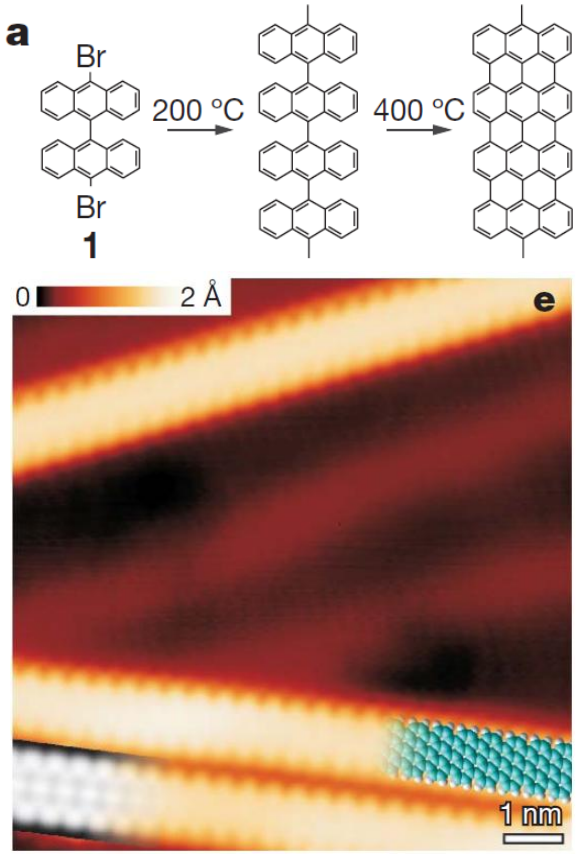


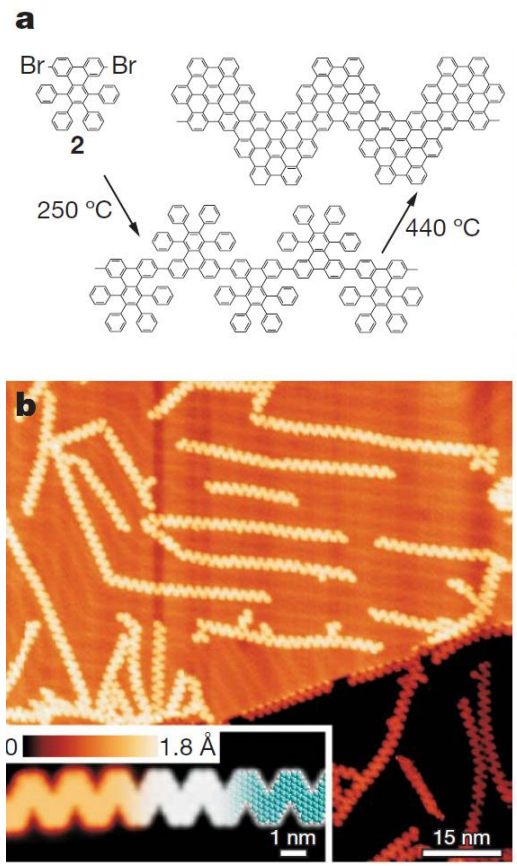


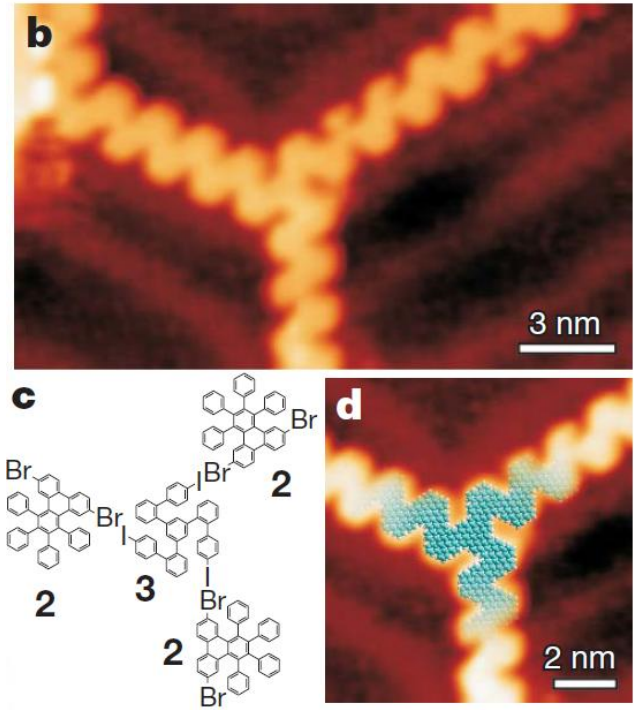
Atomically precise bottom-up fabrication of graphene nanoribbons

Nature **466**, 470 (2010)

Jinming Cai[1]*, Pascal Ruffieux[1]*, Rached Jaafar[1], Marco Bieri[1], Thomas Braun[1], Stephan Blankenburg[1], Matthias Muoth[2], Ari P. Seitsonen[3,4], Moussa Saleh[5], Xinliang Feng[5], Klaus Müllen[5] & Roman Fasel[1,6]

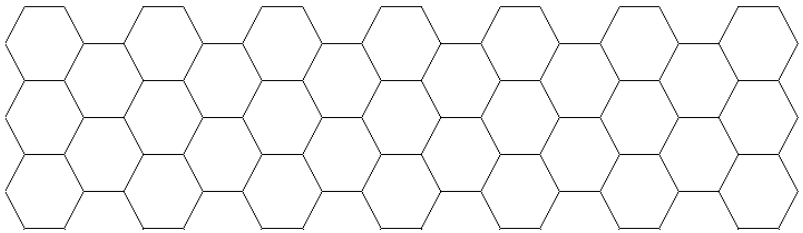




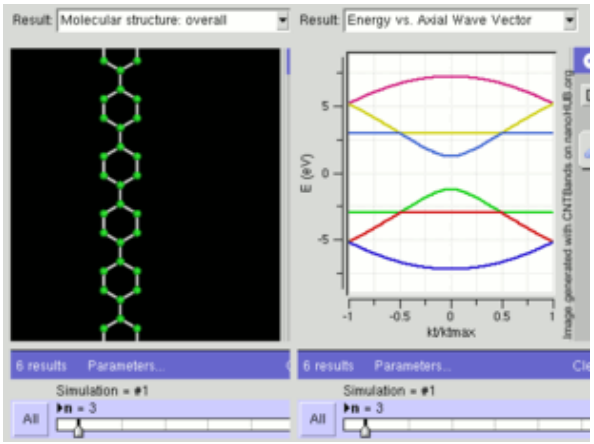


Graphene ribbons

Armchair

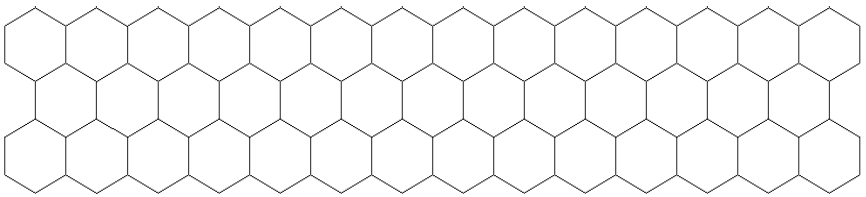


y

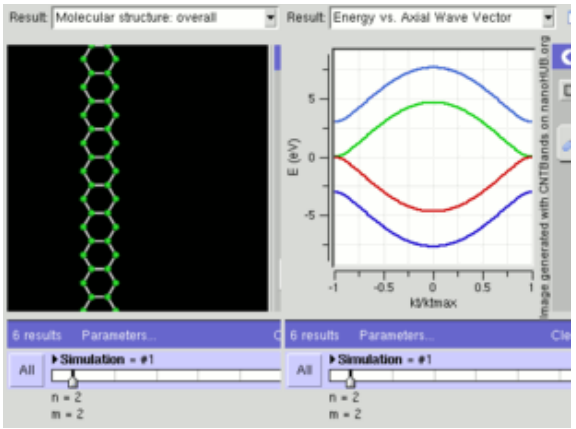


$$E(k_{nx}, k_y)$$

Zigzag



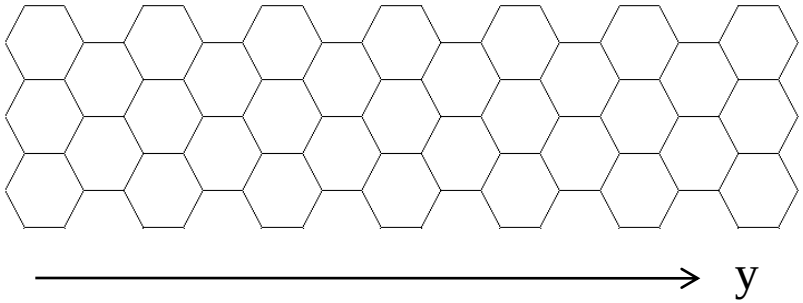
y



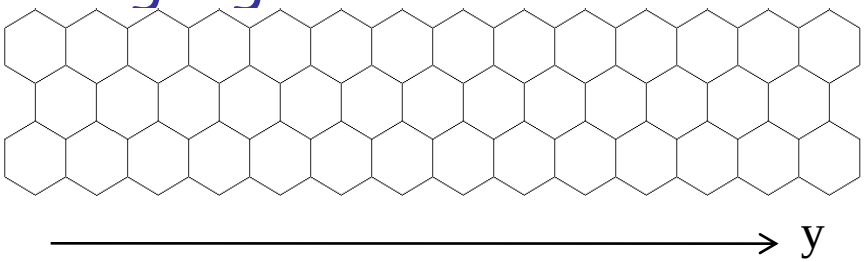
Boundary conditions along x

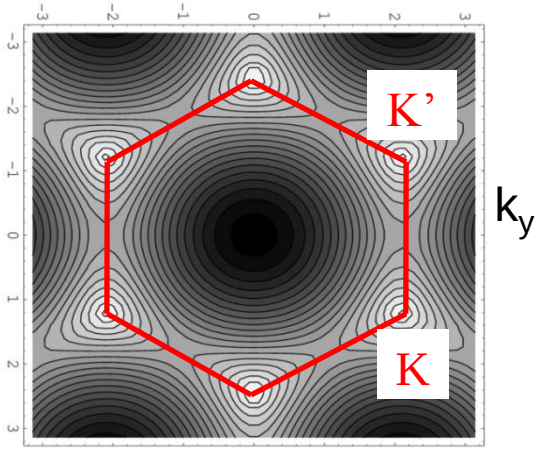
$$\psi(\vec{r}) \propto e^{i\vec{K}\vec{r}} \begin{pmatrix} \varphi_A(x) \\ \varphi_B(x) \end{pmatrix} + e^{i\vec{K}'\vec{r}} \begin{pmatrix} \varphi_A'(x) \\ \varphi_B'(x) \end{pmatrix} \times e^{ik_y y} \longrightarrow \text{Quantization of } k_x$$

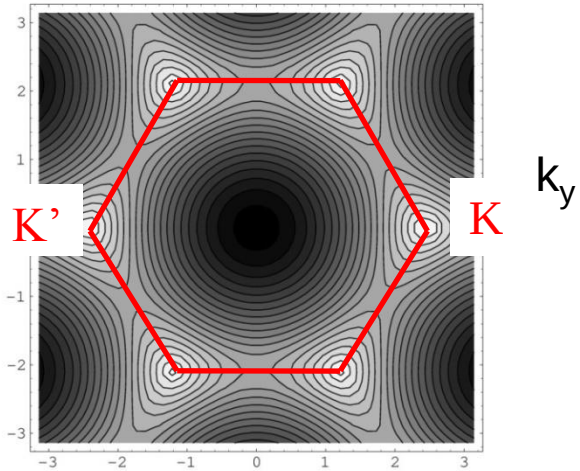
Armchair

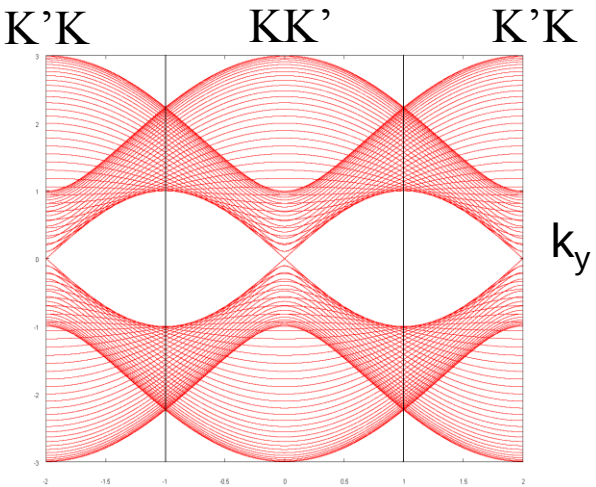


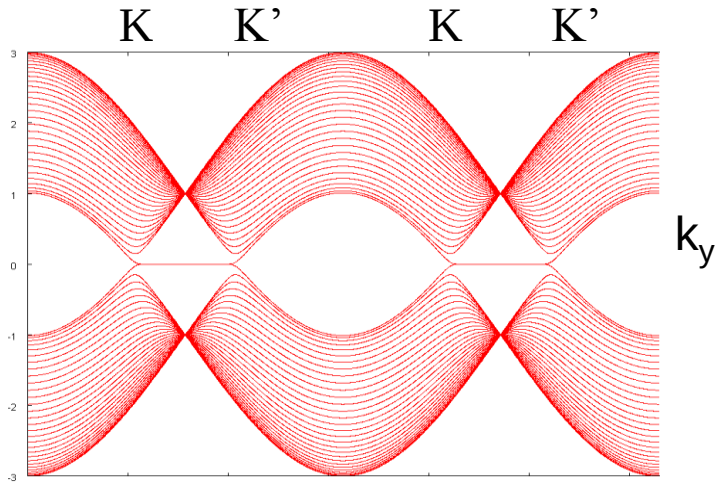
Zigzag

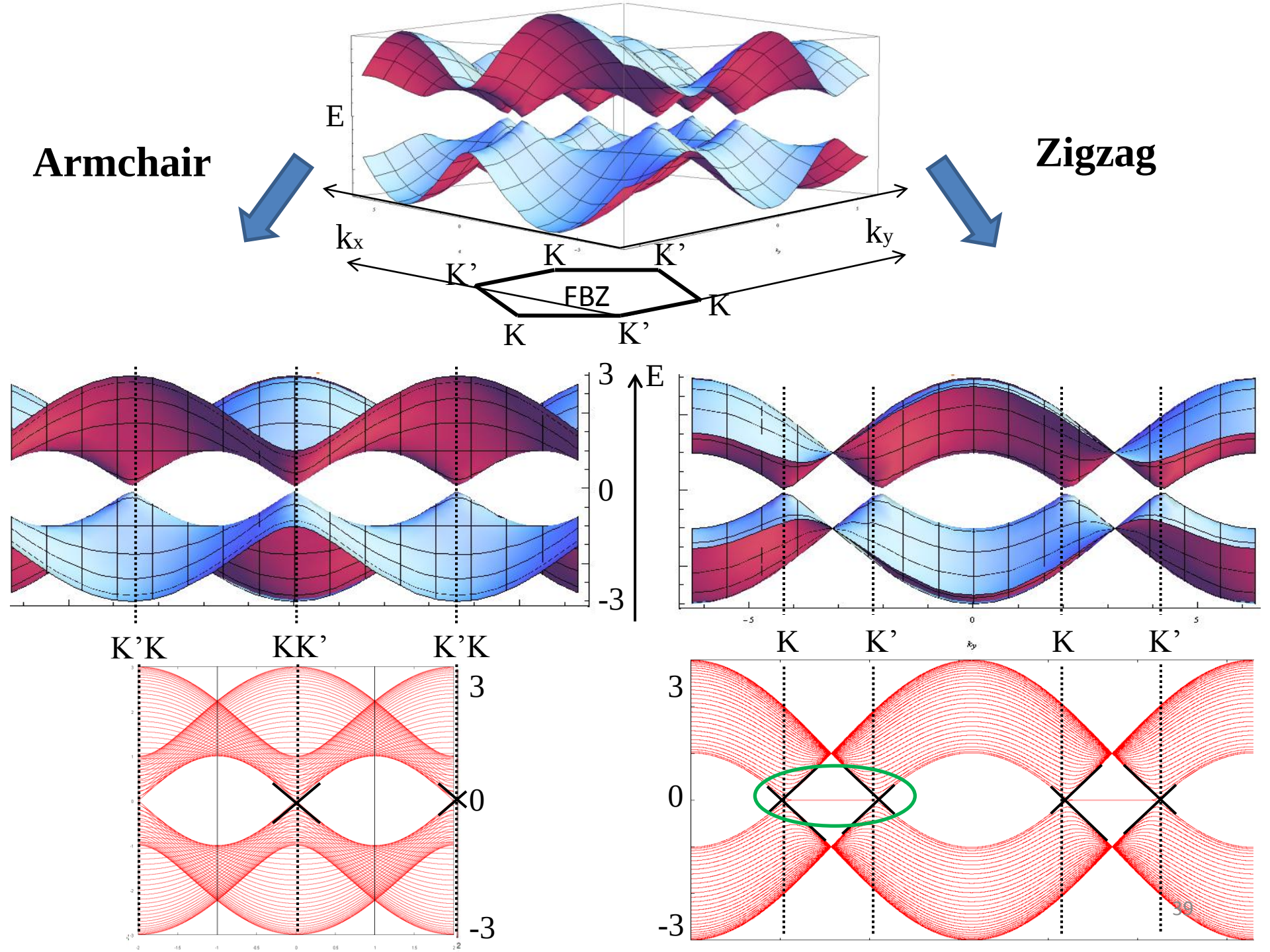






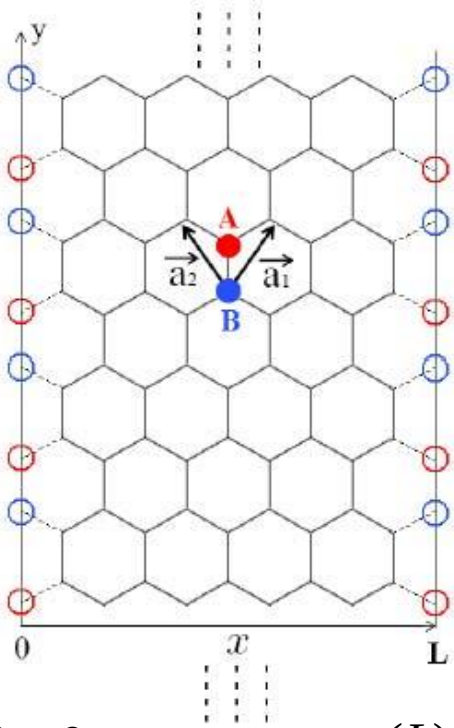





Armchair
Zigzag
E
kx
ky
K
K'
K'
K
K
K'
FBZ
3
0
-3
E
K'K
KK'
K'K
K
K'
K
K'
3
0
-3
3
0
-3

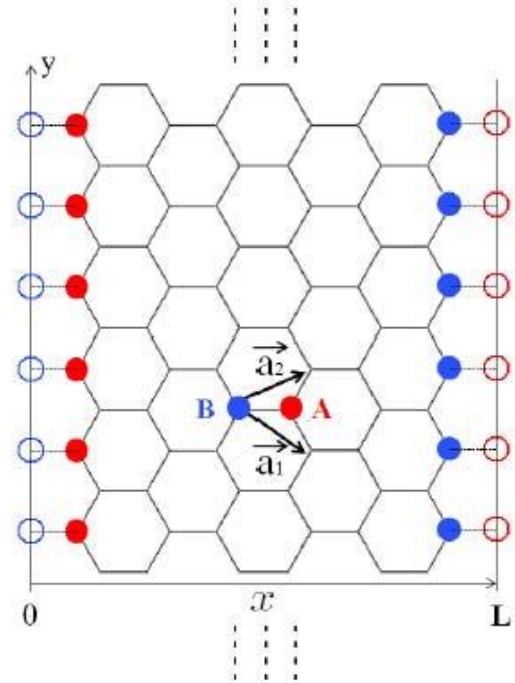
Brey, Fertig, PRB 73, 235411 (2006)

Armchair



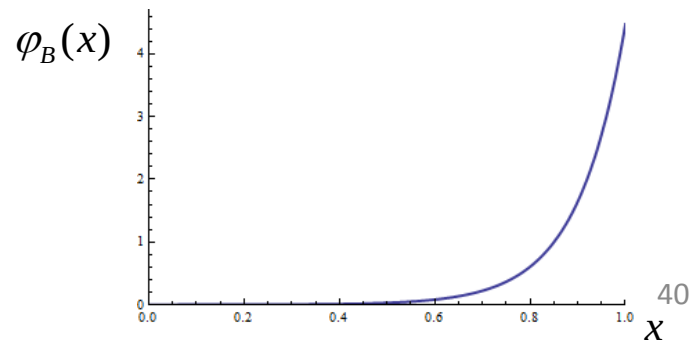
$\psi(0)=0$ $\psi(L)=0$

Zigzag



$\varphi_B(0)=0$ $\varphi_A(L)=0$

Possibility of edge states :



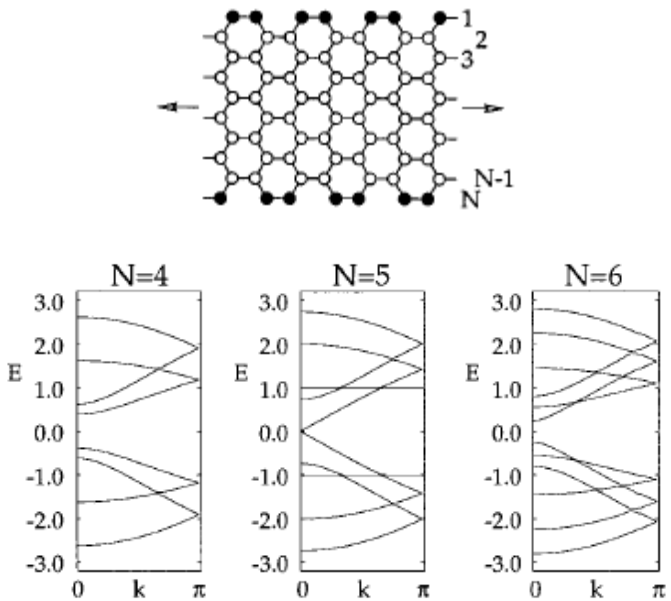
Graphene ribbons

PHYSICAL REVIEW B VOLUME 54, NUMBER 24 15 DECEMBER 1996-II

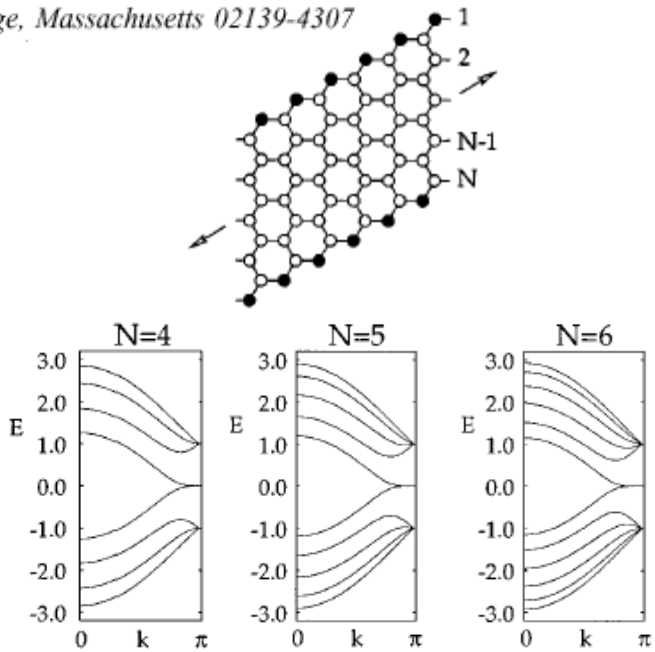
Edge state in graphene ribbons: Nanometer size effect and edge shape dependence

Kyoko Nakada and Mitsutaka Fujita
Institute of Materials Science, University of Tsukuba, Tsukuba 305, Japan

Gene Dresselhaus and Mildred S. Dresselhaus
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139-4307



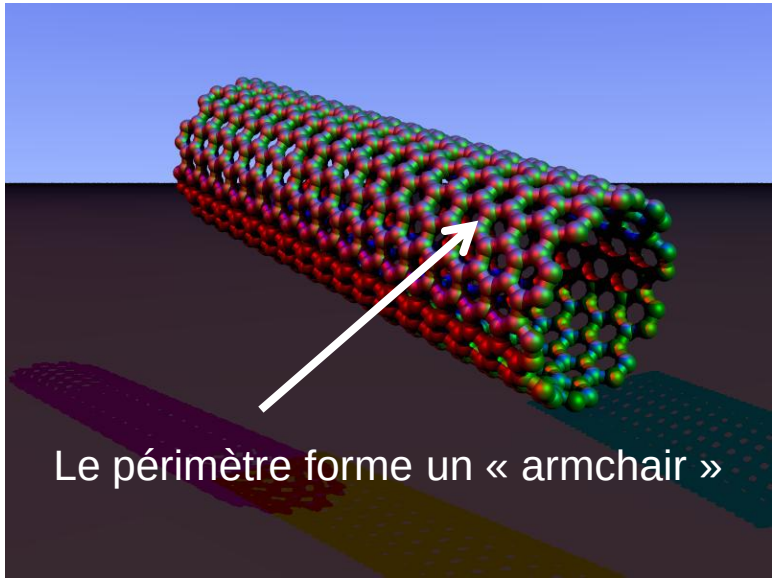
Semiconducting or metallic ribbons

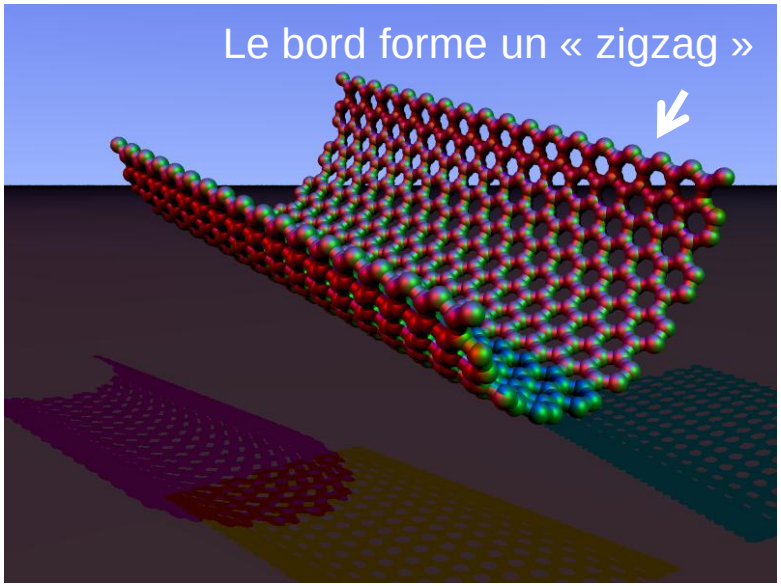


Metallic ribbons

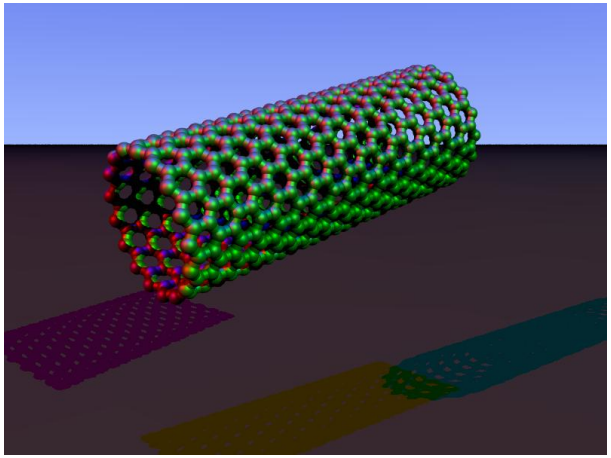
Metallic if N=3n-1

Attention !





Un nanotube « armchair » est obtenu en recollant des bords « zigzag »



Un nanotube « zigzag » est obtenu en recollant des bords « armchair »