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Evidence for Boltzmann's H as a capital eta

Stig Hjalmars

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draws electric and magnetic field lines inside a rectangular waveguide. The dimensions of the waveguide are user variable, as is the frequency of the guided waves, the mode (up to 44), and whether the electric or magnetic field is transverse (TE or TM waves). The waveguide is drawn as a foreshortened two-dimensional image and the program includes full, easy-to-operate rotational capabilities, allowing the waveguide to be viewed from any angle. After having derived expressions for **E** and **H** himself, the student is encouraged to use this program to become familiar with the behavior of these equations and to satisfy himself that the boundary conditions are satisfied. Figure 2 shows a typical representation of the field lines for a TM_{11} mode.

The final program, ARRAYS, shows the horizontal power radiation pattern produced by a linear array of vertical half-wave electric dipole antennas. The number of dipoles, their spacing, and the phase difference of the input to adjacent dipoles are all user variable. Student response to these programs was most favorable when they were used in conjunction with homework problems which would have been difficult to solve without hints at the appropriate time. For example, the class was asked to determine what spacing and phase difference between two dipoles would produce a particular radiation pattern and to use ARRAYS to check the results of their attempts. A representative pattern is shown in Fig. 3. A previous plot will have defined the point of observation and the alignment of the dipoles relative to the coordinate system.

PLANAR, DISCON, WAVGDS, and ARRAYS are written in standard extended FORTRAN IV. Their use requires no programming ability on the part of the student. The only unusual support required for their execution is the Tektronix PLOT-10 Terminal Control System software and the equivalent of a Tektronix 4010 graphic terminal. The authors will be pleased to supply program listings upon request.

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Evidence for Boltzmann's *H* as a capital eta

Stig Hjalmars
Royal Institute of Technology, S-10044 Stockholm, Sweden
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The celebrated "Boltzmann's *H*-theorem," by means of which he gave a gas-kinetic explanation of thermodynamic irreversibility, was first presented in 1872.[1] In this theorem Boltzmann proved that a certain gas-molecular mean value, now universally denoted by *H*, is an always decreasing function, and thus a measure of the negative entropy.

It is, however, a historical puzzle that in 1872 and for more than two decades Boltzmann himself used the symbol *E* instead of the *H*, nowadays so closely connected with his name. The symbol *H* seems to occur for the first time in 1890 in a paper by Burbury, and it was unanimously accepted in 1896, when Boltzmann himself converted to it in his monumental *Vorlesungen über Gastheorie.*[2] The fact that this change was so unusually unanimous, and happened just when the great debate on irreversibility was culminating, cf. the historical review by Brush,[3] shows that the change was not just an isolated question of nomenclature, but was made for some reason, connected with the debate on facts in some interesting way.

The solution of this mystery is hampered by the peculiar lack of any information in the literature about the reason for the change. As yet it has not even been possible to find printed support for the plausible tradition that the meaning of *H* was a capital Greek eta, η, a symbol used by Gibbs for classical entropy since 1873. Certainty on this point would of course be a first step towards the solution of the *H*-puzzle.

In a letter to the Editor of AJP in 1967 Brush[4] challenged the readers "to produce documentary evidence showing that *H* in 'Boltzmann's *H*-theorem' should be capital eta. Presumably, this would have to be a published article, or an unpublished manuscript or letter, dating from the two decades before Boltzmann's death in 1906." This appeal has, however, as yet been unsuccessful (cf. Ref. 3, p. 60, note 14).

Probably, the complete solution of the *H*-puzzle can only be found in the unpublished documents of the time. However, regarding the partial question of the eta interpretation, we will here point out some printed evidence, as yet overlooked, showing that this interpretation was no doubt well known and common among leading scientists of the period.

The first evidence can be found in Boltzmann's *Vorlesungen über Gastheorie*[2] of 1896. When Boltzmann on p. 22 introduces what he refers to as "my so-called *H*-theorem," he makes on this first occasion use of a capital *H* type (boldface sans serif, grotesque), which looks very gaudy and demonstrative in the general typography of the book. The type is not used further in the text, but there can be no doubt that Boltzmann by this striking graphical demonstration in the introduction wanted to show that the meaning was not an ordinary capital *h*. The only reasonable alternative is of course a capital η.

The other evidence is given by Gibbs[5] in his *Elementary Principles in Statistical Mechanics* of 1902. In Chap. XV of this book he treats *inter alia* Boltzmann's H-theorem by means of the grand canonical ensemble. In doing so, he generalizes the η and ψ of his canonical ensemble to H and Ψ, where the mean value of H is Boltzmann's function. He shows on p. 200 that these quantities correspond to the negative entropy and the free energy of thermodynamics. As seen from the way it is introduced, the symbol H is clearly meant as a capital η. This is also definitely verified by the fact that this H, like all other capital Greek letters in the book, is printed vertical (nonslanted), while capital Latin letters are printed with italics (slanted types), as illustrated by the symbols on p. 200, and by the Latin E on p. 158 compared to the Greek E (epsilon) on p. 177. We may also note that Zermelo in his German translation of the book in 1905 also maintains the same classification of the capital letters, although by other typographical means.

The given graphical evidence, of which a detailed account is presented elsewhere,[6] seems to leave no reasonable doubt that during the decade before Boltzmann's death in 1906 at least he himself, Gibbs, and Zermelo meant a capital eta when they wrote H for Boltzmann's function. This suggests that the whole problem of the adoption of the symbol H has a connection with Gibbs and his ensemble-statistical ideas, surely well known to leading scientists long before the publication in 1902.

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When a Gaussian distribution won't do: A short comment on statistics

D. W. Schlitt

Department of Physics and Astronomy, University of Nebraska–Lincoln, Lincoln, Nebraska 68588

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The normal or Gaussian probability distribution rests at the foundation of most of the statistical methods familiar to physicists. The χ^2 test, the F test, Student's t test, etc., each rest on the assumption that some specific quantity has a normal distribution.

What do we do if we know that this assumption of a normal distribution is not valid. A recent paper in this Journal[1] spoke to this question for the case of a Poisson distribution. The author represented the deviation of the Poisson distribution from the normal distribution in terms of a polynomial and made some interesting comments on his result. He did not, however, apply his result to any practical problem in statistics. This is probably because the result is not convenient to use for such things as estimating lifetimes.[2] The purpose of this note is to point out some statistical methods which are convenient to use and to give a simple illustration of the use of one of them.

Statisticians are well aware that not all populations are normally distributed. They have proceeded to develop techniques to use in such cases. Because they often are not discussed in introductory courses in statistics, they have only slowly come into common use in physics.

In order to avoid the assumption of a normal distribution or any specific distribution, a group of methods known as distribution-free or nonparametric methods have been developed; the idea of the robustness of a statistical test has been introduced. A test is robust if it is independent or insensitive to departures from the assumed distribution. A discussion of such tests (directed at physicists) can be found in Ref. 3 and more detailed discussion is given in Ref. 4. The χ^2 test can be replaced by such distribution-free tests as the Cramer–von Mises test or the Kolmogorov–Smirnov test. An example of a recent application of these methods in physics can be found in the work of Ludlam and Slansky.[5]

A typical physics application of statistics is the extraction of an estimate for a physical quantity from experimental data. An example mentioned in Ref. 1 is the determination of a decay constant from counting data. This is an example of the type we are concerned with. We know the number of counts has a Poisson distribution and not a normal distribution. Use of a least-squares or χ^2 method to find the decay constant would immediately suggest itself. As we will see below, these methods are based on an assumption of a normal distribution.

An alternative which can be used if the probability distribution is known, even if it is not normal, is the maximum likelihood method. It is instructive to look at this method in the case of a decay problem. The calculation is simplified version of example 30.20 of Ref. 6. For those interested in a deeper discussion of this problem an early paper by Peierls[7] is of interest. A recent discussion can be found in Ref. 8.

The experiment we consider is one where we count the number of decays in a sample for a time interval Δt_i centered at a time t_i and obtain n_i counts. We do this for N different intervals. The probability of getting n_i counts in the ith interval has a Poisson probability distribution

$$P_i = (n_i!)^{-1}\mu_i^{n_i}\exp(-\mu_i), \quad (1)$$

where μ_i is the expected number of counts. If the decay constant is λ, then the expected number of counts is approximately (assuming no background counts in n_i)

$$\mu_i = N_0\lambda\Delta t_i \exp(-\lambda t_i). \quad (2)$$

We choose this form rather than more exact forms for al-