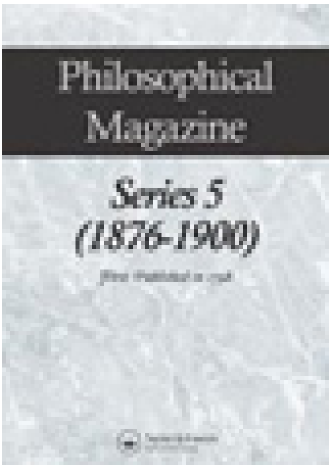

Philosophical Magazine Series 5
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tphm16

LII. On the physical units of nature

G. Johnstone Stoney D.Sc. F.R.S. [a]
[a] Royal Dublin Society
Published online: 08 Jun 2010.

appears at once on putting his case in mathematical form. He supposes two particles, m and m', moving in the same straight line (say the axis of x) with velocities $\frac{dx}{dt}$ and $\frac{dx'}{dt}$, and acting on each other with equal and opposite moving forces F. Then the Force Function, during any time δt, is

$$F\frac{dx}{dt}\delta t - F\frac{dx'}{dt}\delta t.$$

According to Dr. Lodge, this is always $= 0$. But, as is well known, the true equation is

$$F\frac{dx}{dt}\delta t - F\frac{dx'}{dt}\delta t = m\frac{dx}{dt}\frac{d}{dt}\left(\frac{dx}{dt}\right)\delta t - m'\frac{dx'}{dt}\frac{d}{dt}\left(\frac{dx'}{dt}\right)\delta t;$$

where the left-hand side, taken negatively, is the Potential Energy, and the right-hand side is the change in the *vis viva* or Kinetic Energy of the system. This right-hand side clearly varies according to the values of $\frac{dx}{dt}$, $\frac{dx'}{dt}$, or of the velocities. *If these velocities are equal* (assuming the masses also equal), *then* the right-hand side vanishes, and we have as a particular case that which Dr. Lodge assumes to hold universally. What *does* hold universally is that the sum of the two sides (*i. e.* of the Potential Energy and the Kinetic Energy) is constant; and this, and this only, is the principle known as the Conservation of Energy.

WALTER R. BROWNE.

LII. *On the Physical Units of Nature.* *By* G. JOHNSTONE STONEY, *D.Sc., F.R.S., Vice-President of the Royal Dublin Society**.

1. WHEN mathematicians apply the sciences of measurement to the investigation of Nature, they find it convenient to select such units of the several kinds of quantity with which they have to deal as will get rid of any coefficients in their equations which it is possible in this way to avoid. Every advance in our knowledge of Nature enables us to see more distinctly that it would contribute to our further progress if we could effect this simplification, not only with reference to certain classes of phenomena, but throughout the whole domain of Nature.

* From the 'Scientific Proceedings' of the Royal Dublin Society of February 16, 1881, being a paper which had been read before Section A of the British Association, at the Belfast Meeting in 1874. Communicated by the Author.

2. Hitherto the practice of mathematicians has been governed by the demands of the science of mechanics, in the greater part (though not in the whole) of which science it is possible to derive the units of all the other kinds of quantity from any three which may be chosen. A system built in this way upon a foundation which is arbitrarily assumed is necessarily an artificial system. The units which are usually selected as the fundamental units of a series of arbitrary systematic measures are:—

the metre* for lengthine, or unit of length;
the gramme for massine, or unit of mass; and
the solar second for timine, or unit of time.

These three, and all the units which may be derived from them, may be called the metric series of units; and in this investigation they will be represented by small letters. Thus, *the fundamental metric units being*

l_1, the metre, the metric lengthine, or unit of length;
t_1, the solar second, the metric timine, or unit of time;
m_1, the gramme, the metric massine, or unit of mass;

some of the derived units will be:—

v_1, the metric velocitine, or unit of velocity, which is a velocity of one metre per second;

f_1, the metric forcine, or unit of force, which is the hyper-decigramme†—this being the force which, if it acted in a fixed direction on a mass of one gramme for a second,

* Since this paper was written the centimetre has been suggested as a unit of length, and has been largely made use of.

† The hyper-decigramme means the gravitation or downward force towards the earth of a mass which exceeds a decigramme in the ratio of $\frac{10}{g}$, where g is the acceleration of gravity measured in metres per second per second. The appropriateness of the term hyper-decigramme arises from the circumstance that the coefficient $\frac{10}{g}$ everywhere exceeds unity, whether within the earth, outside it, or on its surface; and the convenience of the term arises from the circumstance that *on the earth's surface* the coefficient nowhere exceeds unity by more than a small fraction, so that the hyper-decigramme is a force which but little differs in value from that gravitation or weight of a decigramme with which we, inhabitants of the earth, have become familiar; so that the name suggests to us the *amount* of the force. Gravitation is the downward force, and gravity is the downward acceleration towards the earth as *observed.* They are chiefly due to the attraction of the earth, and in a small degree, when the observation is made on or within the earth, to the earth's rotation. This is the meaning of the word gravity as it is used by the classical writers on mechanics (see Pouillet's *Mécanique, passim*); and the practice of some modern writers, who use this term to designate a force instead of an acceleration, is to be deprecated.

would in that time change its velocity by one metre per second;

m_1, the metric unit of the coefficient in the expression $\mu\frac{mm'}{r^2}$ for the gravitation of two masses towards one another: this unit is the coefficient which should be used if each gramme mass attracted other ponderable matter at a metre distance with such intensity as would impress on it an acceleration towards the attracting gramme of one metre per second per second.

e_1, the electromagnetic electrine, or the electromagnetic unit quantity of electricity in the metric series, is that quantity of each of the two kinds of electricity which must be discharged every second in opposite directions along a wire in order to maintain in it the metric unit current,—this currentine or unit current being defined as the current which must exist in a wire a metre long in order that it may exert a force of a hyper-decigramme on ponderable matter at a metre distance charged with a unit of magnetism; and the unit charge of magnetism of either kind being defined as that quantity which, acting on ponderable matter at a metre distance, charged with an equal quantity of magnetism, exerts on it the unit force—that is, one hyper-decigramme.

3. It is easy to ascertain the relation of this metric electrine to the B.A. (British-Association) standards for electrical measurement, which are those most in use. The B.A. units are electromagnetic units based on the following fundamental units—the second for unit of time, the metre-seven (the quadrant of the earth, or 10^7 metres) for unit of length, and the eleventh-gramme (or gramme divided by 10^{11}) for unit of mass. These were so chosen as to furnish a connected body of systematic units with such values that the practical electrician could conveniently use them. Now the "dimension" of electromagnetic quantity of electricity is $[\mathrm{L}\sqrt{\mathrm{LM}}]$ (see B.A. Report for 1863, p. 159)*. Hence, and from the foregoing values of the lengthine and massine of the B.A. series,

$$e_1 : \text{one Ampère} = 1 : \sqrt{\frac{10^7}{10^{11}}};$$

therefore $e_1 = 100$ Ampères.

* This follows at once from the fundamental equations of electromagnetism, viz.:—

$$\mathrm{F} \propto \frac{\mathrm{EE}^1}{r^2};\quad \mathrm{E} = \mathrm{C}t;\quad \mathrm{F} = \frac{\mathrm{C}l\mathrm{M}}{r^2};\quad \mathrm{F} = \frac{\mathrm{MM}^1}{r^2}.$$

The term Ampère is here used to designate the B.A. unit of quantity corresponding to the ohm (the B.A. electromagnetic unit of resistance), the volt (the corresponding unit of electromotive force), the weber (unit of current), and the farad (unit of capacity). The *electrostatic* units of the B.A. series might with great advantage be called the static-ampère, static-ohm, static-volt, and static-farad.

4. Units like the above, whether of the metric or of the B.A. series, of which three are fundamental, and all others derived from them in such a way as will exclude unnecessary coefficients from our equations, are called systematic units. In forming the existing artificial series of systematic units, it has been usual to regard the units of length, time, and mass as fundamental, and the rest as derived; but *there is nothing to prevent our regarding any three independent members of the series as fundamental, and deriving the others from them.* It is the aim of the present paper to point out that Nature presents us with three such units; and that if we take these as our fundamental units instead of choosing them arbitrarily, we shall bring our quantitative expressions into a more convenient, and doubtless into a more intimate, relation with Nature as it actually exists. I will then approximate to the values of the units of length, time, and mass belonging to this, which is a truly natural series of physical units.

5. For such a purpose we must select phenomena that prevail throughout the whole of Nature, and are not specially associated with individual bodies. The first of Nature's quantities of absolute magnitude to which I will invite attention is that remarkable velocity of an absolute amount, independent of the units in which it is measured, which connects all systematic electrostatic units with the electromagnetic units of the same series. I shall call this velocity V_1. If it were taken as our unit velocity, we should at one stroke have an immense simplification introduced into our treatment of the whole range of electric phenomena, and probably into our study of light and heat.

Again, Nature presents us with one particular coefficient of gravitation, of an absolute amount independent of the units in which it is measured, and which appears to extend to ponderable matter of every description throughout the whole material universe. This coefficient I shall call G_1. If we were to take this as our unit of coefficients of attraction, it is presumable that we might thereby lay the foundation for detecting wherein lies the connexion which we cannot but suspect between this most wonderful property common to all ponderable matter and the other phenomena of nature.

And, finally, Nature presents us, in the phenomenon of electrolysis, with a single definite quantity of electricity which is independent of the particular bodies acted on. To make this clear I shall express "Faraday's Law" in the following terms, which, as I shall show, will give it precision, viz.:—*For each chemical bond which is ruptured within an electrolyte a certain quantity of electricity traverses the electrolyte, which is the same in all cases.* This definite quantity of electricity I shall call E_1. If we make this our unit quantity of electricity, we shall probably have made a very important step in our study of molecular phenomena.

Hence we have very good reason to suppose that in V_1, G_1, and E_1 we have three of a series of systematic units that in an eminent sense are the units of nature, and stand in an intimate relation with the work which goes on in her mighty laboratory.

6. The approximate values of V_1 and G_1 are known; and I will presently endeavour to evaluate E_1. V_1 has been variously determined by experiment as 3·10 metre-eights per second, 2·82 metre-eights per second, 2·88 metre-eights per second, and may be assumed to be not far from 3 metre-eights per second. Accordingly we may put

$$V_1 = 3 \text{ VIII metres per second.} \quad . \quad . \quad . \quad (1)$$

Similarly, if we use the value given by Sir John Herschel for the mass of the earth, viz. 5942 XVIII* English tons, which = XXIV grammes, we find that

$$G_1 = \frac{2}{3}\frac{1}{\text{XIII}}[\mu]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

7. To determine E_1, we must first establish a relation between the gaseous molecule of a body and what in chemistry is called its atom. To do this, let us start with the definition that a *chemical atom* is the smallest mass of each kind of ponderable matter that *has been found* to enter or leave a combination. Now, from Boyle and Charles's law we know that in all gases there are approximately the same number of molecules per litre, if they be taken at the same temperature and pressure; from experiments on diffusion we know that these molecules are alike in mass; and from the phenomena of chemistry we know that they are alike in other respects.

* The Roman figures following a number stand for cyphers. Thus, 3 VIII signifies 3×10^8, and 5942 XVIII stands for eighteen cyphers following 5942. Where no number precedes the Roman figures the number 1 is to be understood; so that in XXIV grammes, the Roman figures stand for 1 followed by 24 cyphers, in other words, for 10^{24}, a number which may conveniently be called the unit-twenty-four.

Let, then, a litre of hydrogen and a litre of chlorine be mixed and exploded, and let the resulting hydrochloric-acid gas be brought back to the original temperature and pressure. It is then found to measure two litres. Hence, if N be the number of molecules in a litre of gas at that temperature and pressure, we learn by this experiment that N molecules of hydrogen and N molecules of chlorine produce 2N molecules of hydrochloric acid. Hence, and since the molecules within each gas are alike, each molecule of hydrochloric acid must contain the quantity of hydrogen represented by a semi-molecule of hydrogen gas and the quantity of chlorine represented by a semi-molecule of chlorine gas. We are thus introduced to the semi-molecule of each of these gases as a quantity which enters into combination; and as no other experiments suggest a smaller quantity, the semi-molecule of hydrogen and the semi-molecule of chlorine are, in the present state of science, to be accepted as the chemical atoms of these substances. Hence we may write

H, the atom of hydrogen = the semi- gaseous molecule of hydrogen; and
Cl, the atom of chlorine = the semi- gaseous molecule of chlorine;

and we see that HCl is the proper formula for hydrochloric acid. We may further deduce from the observed densities of the gases, that the masses of the atoms of hydrogen, chlorine, and hydrochloric acid are to one another in the ratio of 1, $35\frac{1}{2}$, $36\frac{1}{2}$.

Another experiment shows us that a litre of steam may be resolved into a litre of hydrogen and half a litre of oxygen at the same temperature and pressure—in other words, that N molecules of steam are formed of N molecules of hydrogen and $\frac{N}{2}$ molecules of oxygen. Hence each molecule of steam contains a whole molecule (or two atoms) of hydrogen and a semi-molecule of oxygen. We thus arrive at the semi-molecule of oxygen as a quantity that enters into combination; and as all other experiments with oxygen concur, the semi-molecule of oxygen is to be received as its atom, and H_2O is the proper formula for what is both the gaseous molecule and the atom of water. From the densities we may also deduce that 16 is the atomic weight of oxygen, *i. e.* that an atom of oxygen is sixteen times as heavy as an atom of hydrogen.

Similarly from the densities of ammonia and of its constituents, we learn that the atom of nitrogen is the semi-molecule, and that the mass of its atom is fourteen times that of hydrogen.

It must not be assumed that the atom is always the semi-molecule. In some cases it is found to be the entire molecule, and in other cases the quarter molecule. Thus the mercuric compounds of mercury give vapours of the same bulk as the vapour of the mercury they contain, and indicate an atom of mercury equal to its molecule; while the other volatile compounds of mercury contain more than one molecule of mercury in each molecule of the compound, and therefore do not disturb this conclusion. Again, a litre of phosphuretted hydrogen yields a quarter of a litre of the vapour of phosphorus and one and a half litre of hydrogen, indicating that the quarter molecule of phosphorus is its atom. The same is true of arsenic.

A similar treatment of marsh-gas furnishes 12 as the mass of an atom of carbon, although carbon is not sufficiently volatile to enable us to ascertain the relation of its atom to its gaseous molecule.

By extending this method to all the available cases, we may deduce *from the fundamental properties of gases* a demonstration of a great part of the modern Table of atomic weights and of the doctrine of atomicity which depends on it. Thus, two bonds * are necessary to connect the group SO_4 with the two atoms of hydrogen that are united to it in sulphuric acid, while one bond is sufficient to join the atoms of hydrogen and chlorine in an atom of hydrochloric acid, and so in other cases.

8. Now the whole of the quantitative facts of electrolysis may be summed up in the statement that A DEFINITE QUANTITY OF ELECTRICITY TRAVERSES THE SOLUTION FOR EACH BOND THAT IS SEPARATED. Thus, if a current pass in succession through vessels containing solutions of sulphuric acid and hydrochloric acid, two atoms of hydrochloric acid will be decomposed in the one vessel for each atom of sulphuric acid that is decomposed in the other; but *the number of bonds separated will be the same in both vessels.*

9. It is the quantity of electricity that passes per bond separated that we have now to determine, and this may be done approximately in the following manner. Several inquiries (see Prof. J. Loschmidt, "Zur Grösse der Luftmolecüle," Academy of Vienna, Oct. 1865; G. Johnstone Stoney, "On the Internal Motions of Gases," Phil. Mag. August 1868; and Sir William Thomson, "On the Size of Atoms," 'Nature,' March 31, 1870) have led up to the conclusion that the number of molecules in each cubic millimetre of a gas at

* The word bond is here used of the *connexions* between atoms when they enter into combination. When we use this, which seems the proper signification of the word, the bonds are to be distinguished from the hands or feelers which each atom has, and which, by grappling with the hands or feelers of other atoms, establish bonds between them.

atmospheric temperatures and pressures is somewhere about a unit-eighteen (10^{18}). Hence the number of molecules in a litre will be about a unit-XXIV. Now, a litre of hydrogen at atmospheric pressures and temperatures weighs, roughly speaking, a decigramme. Hence the mass of each molecule of hydrogen is a quantity of the same order as a decigramme divided by a unit-XXIV, *i. e.* a XXVth gramme. The chemical atom is half of this. Hence the mass of a chemical atom of hydrogen may be taken to be somewhere about half a twenty-fifth-gramme. There is no advantage in retaining the coefficient half in an estimate in which we are not even sure that we have assigned the correct power of 10; and I will therefore, for the sake of simplicity, take the XXVth gramme as being such an approach as we can attempt to the value of the mass of an atom of hydrogen.

10. Now, it has been ascertained by experiment that, for every ampère of electricity that passes, ninety-two sixth-grammes (*i. e.* ninety-two millionths of a gramme of water) are decomposed (see Brit.-Assoc. Report, 1863, p. 160). This water is the result of a secondary action in the voltameter; but that does not affect the present inquiry. Ninety-two VIth grammes of water contain about one Vth gramme of hydrogen, which is therefore the quantity evolved. The metric unit of electricity e_1 is 100 ampères, and will therefore set free 100 Vth grammes of hydrogen, *i. e.* one milligramme. Now it appears, from the last paragraph, that this quantity of hydrogen contains $\frac{\text{XXV}}{1000}$ atoms, *i. e.* XXII atoms. And as there is a bond ruptured for each atom of hydrogen set free, this is also the number of bonds broken; in other words, the quantity of electricity corresponding to each chemical bond separated is

$$E_1 = \frac{1}{\text{XXII}} e_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Collecting our numerical results, they are

$$V_1 = 3 \text{ VIII metres per second}, \quad . \quad . \quad (1)$$

$$G_1 = \frac{2}{3} \frac{\mu_1}{\text{XIII}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$E_1 = \frac{e_1}{\text{XXII}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$= \frac{1}{\text{XX}} \text{ ampère.}$$

We have thus obtained approximate values in known measures for the three great fundamental units offered to us by Nature, upon which may be built an entire series of physical units deserving of the title of a truly Natural Series of Physical Units.

11. It now only remains to deduce the units of length, time, and mass belonging to this series. For this purpose we may use dimensional equations. Remembering, as is well known, that the dimension of a unit of velocity is $\left[\frac{L}{T}\right]$, that of a unit of coefficients of attraction $\left[\frac{L^3}{MT^2}\right]$, and that of an electromagnetic unit of quantity $[\sqrt{LM}]$, we find from equations (1), (2), and (3) respectively that

$$\frac{L_1}{T_1} = A\frac{l_1}{t_1}, \quad \ldots \ldots \quad (4)$$

$$\frac{L_1^3}{M_1T_1^2} = B\frac{l_1}{m_1t_1^2}, \quad \ldots \ldots \quad (5)$$

$$\sqrt{L_1M_1} = C\sqrt{l_1m_1}\,; \quad \ldots \ldots \quad (6)$$

in which L_1, M_1, and T_1 are used to designate the units of length, mass, and time in the "Natural" series; while l_1, m_1, and t_1 represent the corresponding units in the metric series, viz. the metre, gramme, and second. A, B, and C also are used, for brevity, to stand for the numerical coefficients of equations (1), (2), and (3); viz. for the numbers 3 VIII, $\frac{2}{3}\frac{1}{\text{XIII}}$, and $\frac{1}{\text{XXII}}$.

Solving equations (4), (5), and (6), we find

$$L_1 = \frac{C\sqrt{B}}{A}\,l_1, \quad \ldots \ldots \quad (7)$$

$$T_1 = \frac{C\sqrt{B}}{A^2}\,t_1, \quad \ldots \ldots \quad (8)$$

$$M_1 = \frac{CA}{\sqrt{B}}\,m_1. \quad \ldots \ldots \quad (9)$$

Substituting for A and B their numerical values, and writing metre, second, and gramme for l_1, t_1, m_1,

$$L_1 = C\;\frac{1}{3\sqrt{15}}\,\frac{1}{\text{XIV}}\text{ metres},$$

$$T_1 = C\frac{1}{3}\,\frac{1}{3\sqrt{15}}\,\frac{1}{\text{XXII}}\text{ seconds},$$

$$M_1 = C\;\;3\sqrt{15}\text{ XIV grammes}\,;$$

or, more simply (inasmuch as 10 is sufficiently near to $3\sqrt{15}$ to be used instead of it in an approximation like the present),

$$L_1 = C \frac{1}{\text{XV}} \text{ metres}, \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$T_1 = C\frac{1}{3}\frac{1}{\text{XXIII}} \text{ seconds}, \quad . \quad . \quad . \quad (11)$$

$$M_1 = C \text{ XV grammes}. \quad . \quad . \quad . \quad . \quad (12)$$

In obtaining these equations we have only used the numerical values of V_1 and G_1, which are known to a satisfactory degree of approximation; and if we go no further, there will remain but one arbitrary member in the entire of the resulting series of systematic physical units.

12. If we also introduce the numerical value found above for C, which depends on E_1 and is less accurately known, we obtain the following actual values for these units of Nature:—

$$L_1 = \frac{1}{\text{XXXVII}} \text{ of a metre}; \quad . \quad . \quad . \quad (13)$$

$$T_1 = \frac{1}{3}\frac{1}{\text{XLV}} \text{ of a second}; \quad . \quad . \quad . \quad (14)$$

$$M_1 = \frac{1}{\text{VII}} \text{ of a gramme}. \quad . \quad . \quad . \quad (15)$$

Or, in other words—

The natural unit of length approaches in value to the thirty-seventh metre (*i. e.* the metre divided by 10^{37}).

The natural unit of time approaches in value to one third of the forty-fifth second (*i. e.* one third of the second of time divided by 10^{45}); and

The natural unit of mass approaches to the seventh gramme (*i. e.* the gramme divided by 10^7).

13. This appears the best attempt we can yet make to determine these remarkable units. In the series to which they belong all the electrostatic units will be identical with the corresponding electromagnetic units, all the forces of Nature that are known to obey the law of the inverse square, whether they arise from gravitation, electricity, or magnetism, will be expressed without coefficients, and the chemical bond, which seems to be the unit of concrete Nature, is brought into its proper relation to physics.

Postscript.—Many persons find it difficult to conceive of G_1 as a unit. G_1 may be avoided and M_1 be substituted for it, if M_1 be defined as a mass such that it attracts an equal mass at a distance with the same force with which two units of electricity, as defined above in section 5 (*i. e.* each equal to E_1), would, if placed at the same distance asunder, act on each other. The three fundamental units of the Natural System will then be V_1, E_1, and M_1, from which all others are to be derived. This M_1 is the same as the M_1 of sections 11 and 12.