

Nobel Lecture

Nobel Lecture, June 2, 1920

The Genesis and Present State of Development of the Quantum Theory

If I take it correctly that the duty imposed upon me today is to give a public lecture on my writings, then I believe that this task, the importance of which I am well aware through the gratitude felt towards the noble-minded founder of our Foundation, cannot be more suitably fulfilled than by my trying to give you the story of the origin of the quantum theory in broad outlines and to couple with this, a picture in a small frame, of the development of this theory up to now, and its present-day significance for physics.

When I look back to the time, already twenty years ago, when the concept and magnitude of the physical quantum of action began, for the first time, to unfold from the mass of experimental facts, and again, to the long and ever tortuous path which led, finally, to its disclosure, the whole development seems to me to provide a fresh illustration of the long-since proved saying of Goethe's that man errs as long as he strives. And the whole strenuous intellectual work of an industrious research worker would appear, after all, in vain and hopeless, if he were not occasionally through some striking facts to find that he had, at the end of all his criss-cross journeys, at last accomplished at least one step which was conclusively nearer the truth. An indispensable hypothesis, even though still far from being a guarantee of success, is however the pursuit of a specific aim, whose lighted beacon, even by initial failures, is not betrayed.

For many years, such an aim for me was to find the solution to the problem of the distribution of energy in the normal spectrum of radiating heat. Since Gustav Kirchhoff has shown that the state of the heat radiation which takes place in a cavity bounded by any emitting and absorbing substances of uniform temperature is entirely independent upon the nature of the substances, a universal function was demonstrated which was dependent only upon temperature and wavelength, but in no way upon the properties of any substance. And the discovery of this remarkable function promised deeper insight into the connection between energy and temperature which is, in fact, the major problem in thermodynamics and thus in the whole of molecular physics. To attain this there was no other way but to seek out from all the different substances existing in Nature one of known emissive and absorptive power, and to calculate the properties of the heat radiation in stationary energy exchange with it. According to Kirchhoff's Law, this would have to prove independent of the nature of the body.

Heinrich Hertz's linear oscillator, whose laws of emission, for a given frequency, Hertz had just previously completely developed, seemed to me to be a particularly suitable device for this purpose. If a number of such Hertzian oscillators are set up within a cavity surrounded by a sphere of reflecting walls, then by analogy with audio oscillators and resonators, energy will be exchanged between them by the output and absorption of electromagnetic waves, and finally stationary radiation corresponding to Kirchhoff's Law, the so-called black-body

radiation, should be set up within the cavity. I was filled at that time with what would be thought today naively charming and agreeable expectations, that the laws of classical electrodynamics would, if approached in a sufficiently general manner with the avoidance of special hypotheses, be sufficient to enable us to grasp the most significant part of the process to be expected, and thus to achieve the desired aim. I, therefore, developed first the laws of emission and absorption of a linear resonator on the most general basis, in fact I proceeded on such a detour which could well have been avoided had I made use of the existing electron theory of [H.A. Lorentz](#), already basically complete. But since I did not quite trust the electron hypothesis, I preferred to observe that energy which flowed in and out through an enclosing spherical surface around the resonator at a suitable distance from it. By this method, only processes in a pure vacuum came into account, but a knowledge of these was sufficient to draw the necessary conclusions however, about the energy changes in the resonator.

The fruit of this long series of investigations, of which some, by comparison with existing observations, mainly the vapour measurements by V. Bjerknes, were susceptible to checking, and were thereby confirmed, was the establishment of the general connection between the energy of a resonator of specific natural period of vibration and the energy radiation of the corresponding spectral region in the surrounding field under conditions of stationary energy exchange. The noteworthy result was found that this connection was in no way dependent upon the nature of the resonator, particularly its attenuation constants - a circumstance which I welcomed happily since the whole problem thus became simpler, for instead of the energy of radiation, the energy of the resonator could be taken and, thereby, a complex system, composed of many degrees of freedom, could be replaced by a simple system of one degree of freedom.

Nevertheless, the result meant no more than a preparatory step towards the initial onslaught on the particular problem which now towered with all its fearsome height even steeper before me. The first attempt upon it went wrong, for my original secret hope that the radiation emitted from the resonator can be in some characteristic way or other distinguished from the absorbed radiation and thereby allow a differential equation to be set up, from the integration of which one could gain some special condition for the properties of stationary radiation, proved false. The resonator reacted only to those rays which it also emitted, and was not in the slightest bit sensitive to the adjacent spectral regions.

Furthermore, my hypothesis that the resonator could exercise a unilateral, i.e. irreversible, effect upon the energy in the surrounding radiation field, was strongly contested by Ludwig Boltzmann, who, with his riper experience in these problems, proved that according to the laws of classical dynamics each of the processes observed by me can proceed in exactly the opposite direction, in such a way, that a spherical wave emitted from the resonator, returns and contracts in steadily diminishing concentric spherical surfaces inwards to the resonator, and is again absorbed by it, thereby allowing the formerly absorbed energy to be re-transmitted into space in the direction from which it came. And when I excluded this kind of singular process, such as an inwardly directed wave, by means of the introduction of a limiting definition, the hypothesis of natural radiation, all these analyses still showed ever more clearly that an important connecting element or term, essential for the complete grasp of the core of the problem, must be missing.

So there was nothing left for me but to tackle the problem from the opposite side, that of thermodynamics, in which field I felt, moreover, more confident. In fact my earlier studies of the Second Law of Heat Theory stood me in good stead, so that from the start I tried to get a

connection, not between the temperature but rather the entropy of the resonator and its energy, and in fact, not its entropy exactly but the second derivative with respect to the energy since this has a direct physical meaning for the irreversibility of the energy exchange between resonator and radiation. Since I was, however, at that time still too far oriented towards the phenomenological aspect to come to closer quarters with the connection between entropy and probability, I saw myself, at first, relying solely upon the existing results of experience. In the foreground of interest at that time, in 1899, was the energy distribution law established by [W. Wien](#) shortly before, whose experimental proof was taken up, on the one hand, by F. Paschen at the Technische Hochschule in Hannover, and, on the other hand, by O. Lummer and E. Pringsheim at the State Institution in Charlottenburg. This law brought out the dependence of the radiation intensity on the temperature, representing it by an exponential function. If one calculates the connection between the entropy and the energy of a resonator, determined by the above law, the remarkable result is obtained that the reciprocal value of the above-mentioned differential coefficient, which I will call R , is proportional to the energy. This extremely simple relationship can be considered as the completely adequate expression of Wien's energy distribution law; for with the dependence upon the energy, the dependence upon the wavelength is always directly given through the general, well-established displacement law by Wien.

Since the whole problem concerned a universal law of Nature, and since at that time, as still today, I held the unshakeable opinion that the simpler the presentation of a particular law of Nature, the more general it is - though at the same time, which formula to take as the simpler, is a problem which cannot always be confidently and finally decided - I believed for a long time that the law that the quantity R is proportional to the energy, should be looked upon as the basis for the whole energy distribution law. This concept could not be maintained for long in the face of fresh measurements. Whilst for small values of the energy and for short waves, Wien's law was satisfactorily confirmed, noteworthy deviations for larger wavelengths were found, first by O. Lummer and E. Pringsheim, and finally by H. Rubens and F. Kurlbaum, whose measurements on the infrared residual rays of fluorite and rock salt revealed a totally different, though still extremely simple relationship, characterized by the fact that the quantity R is not proportional to the energy, but to the square of the energy, and in fact this holds with increasing accuracy for greater energies and wavelengths.

So, through direct experiment, two simple limits were determined for the function R : for small energies, proportionality with the energy; for greater energies, proportionality with the square of the energy. There was no better alternative but to make, for the general case, the quantity R equal to the sum of two terms, one of the first power, and one of the second power of the energy, so that for small energies the first is predominant, whilst for the greater energies the second is dominant. Thus the new radiation formula was found, which, in the face of its experimental proof, has stood firm to a reasonable extent until now. Even today, admittedly, we cannot talk of final exact confirmation. In fact, a fresh attempt at proof is urgently required.

However, even if the radiation formula should prove itself to be absolutely accurate, it would still only have, within the significance of a happily chosen interpolation formula, a strictly limited value. For this reason, I busied myself, from then on, that is, from the day of its establishment, with the task of elucidating a true physical character for the formula, and this problem led me automatically to a consideration of the connection between entropy and probability, that is, Boltzmann's trend of ideas; until after some weeks of the most strenuous

work of my life, light came into the darkness, and a new undreamed-of perspective opened up before me.

I must make a small intercalation at this point. According to Boltzmann, entropy is a measure for physical probability, and the nature and essence of the Second Law of Heat Theory is that in Nature a state occurs more frequently, the more probable it is. Now one always measures in Nature the difference in entropies, never the entropy itself, and to this extent one cannot speak of the absolute entropy of a state, without a certain arbitrariness. Nevertheless, it is useful to introduce the suitably defined absolute value of entropy, namely for the reason that with its help certain general laws can be particularly easily formulated. The case seems to be parallel, as I see it, with that of energy. Energy itself cannot be measured, only its difference. For that reason one used to deal, not with energy, but with work, and even Ernst Mach, who had so much to do with the Law of Conservation of Energy, and who in principle kept away from all speculations beyond the field of observation, has always avoided speaking of energy itself. Likewise, in thermochemistry, one has always stuck to the thermal effect, that is, to energy differences, until [Wilhelm Ostwald](#) in particular emphatically showed that many detailed considerations could be significantly abbreviated if one dealt with energy itself instead of with calorimetric numbers. The additive constant which was at first still undetermined in the expression for energy, has later been finally determined through the relativistic law of the proportionality between energy and inertia.

In a similar way to that for energy, an absolute value can be defined also for entropy and, as a result thereof, for the physical probability too, e.g. by so fixing the additive constant that energy and entropy disappear together. On the basis of a consideration of this kind a specific, relatively simple combinatorial method was obtained for the calculation of the physical probability of a specified energy distribution in a system of resonators, which led exactly to that entropy expression determined by the radiation law, and it brought me much-valued satisfaction for the many disappointments when Ludwig Boltzmann, in the letter returning my essay, expressed his interest and basic agreement with the train of thoughts expounded in it.

For the numerical treatment of the indicated consideration of probability, knowledge of two universal constants is required, both of which have an independent physical meaning, and whose subsequent evaluation from the law of radiation must provide proof as to whether the whole method is to be looked upon as a mere artifice for calculation, or whether it has an inherent real physical sense and interpretation. The first constant is of a more formal nature and is connected with the definition of temperature. If temperature were to be defined as the average kinetic energy of a molecule in an ideal gas, that is, as a tiny, little quantity, then the constant would have the value $2/3$. In conventional temperature measure, on the contrary, the constant has an extremely small value which stands, naturally, in close association with the energy of a single molecule, and an exact knowledge of which leads, therefore, to the calculation of the mass of a molecule and those parameters related to it. This constant is often referred to as Boltzmann's constant, although, to my knowledge, Boltzmann himself never introduced it - a peculiar state of affairs, which can be explained by the fact that Boltzmann, as appears from his occasional utterances, never gave thought to the possibility of carrying out an exact measurement of the constant. Nothing can better illustrate the positive and hectic pace of progress which the art of experimenters has made over the past twenty years, than the fact that since that time, not only one, but a great number of methods have been discovered for measuring the mass of a molecule with practically the same accuracy as that attained for a planet.

At the time when I carried out the corresponding calculation from the radiation law, an exact proof of the number obtained was quite impossible, and not much more could be done than to determine the order of magnitude which was admissible. It was shortly afterward that [E. Rutherford](#) and H. Geiger succeeded in determining, by direct counting of the alpha particles, the value of the electrical elementary charge, which they found to be 4.65×10^{-10} electrostatic units; and the agreement of this figure with the number calculated by me, 4.69×10^{-10} , could be taken as decisive confirmation of the usefulness of my theory. Since then, more sophisticated methods have led to a slightly higher value, these measurements being carried out by E. Regener, [R.A. Millikan](#), and others.

The explanation of the second universal constant of the radiation law was not so easy. Because it represents the product of energy and time (according to the first calculation it was 6.55×10^{-27} erg sec), I described it as the elementary quantum of action. Whilst it was completely indispensable for obtaining the correct expression for entropy - since only with its help could the magnitude of the "elementary regions" or "free rooms for action" of the probability, decisive for the assigned probability consideration, be determined - it proved elusive and resistant to all efforts to fit it into the framework of classical theory. As long as it was looked upon as infinitely small, that is, for large energies or long periods of time, everything went well; but in the general case, however, a gap yawned open in some place or other, which was the more striking, the weaker and faster the vibrations that were considered. The foundering of all efforts to bridge the chasm soon left little doubt. Either the quantum of action was a fictional quantity, then the whole deduction of the radiation law was in the main illusory and represented nothing more than an empty non-significant play on formulae, or the derivation of the radiation law was based on a sound physical conception. In this case the quantum of action must play a fundamental role in physics, and here was something entirely new, never before heard of, which seemed called upon to basically revise all our physical thinking, built as this was, since the establishment of the infinitesimal calculus by Leibniz and Newton, upon the acceptance of the continuity of all causative connections.

Experiment has decided for the second alternative. That the decision could be made so soon and so definitely was due not to the proving of the energy distribution law of heat radiation, still less to the special derivation of that law devised by me, but rather should it be attributed to the restless forwardthrusting work of those research workers who used the quantum of action to help them in their own investigations and experiments. The first impact in this field was made by [A. Einstein](#) who, on the one hand, pointed out that the introduction of the energy quanta, determined by the quantum of action, appeared suitable for obtaining a simple explanation for a series of noteworthy observations during the action of light, such as Stokes' Law, electron emission, and gas ionization, and, on the other hand, derived a formula for the specific heat of a solid body through the identification of the expression for the energy of a system of resonators with that of the energy of a solid body, and this formula expresses, more or less correctly, the changes in specific heat, particularly its reduction with falling temperature. The result was the emergence, in all directions, of a number of problems whose more accurate and extensive elaboration in the course of time brought to light a mass of valuable material. I cannot give here even an approximate report on the abundance of the work carried out. Only the most important and characteristic steps along the path of progressive knowledge can be high-lighted here.

First come thermal and chemical processes. As far as the specific heat of solid bodies is concerned, Einstein's theory, which rested upon the assumption of a single natural vibration of the atom, was extended by M. Born and Th. von Kármán to the case of various kinds of

natural vibrations, which approached more nearly to the truth. P. Debye succeeded, by means of a bold simplification of the stipulations for the character of natural vibrations, in producing a relatively simple formula for the specific heat of solid bodies which, particularly for low temperatures, not only satisfactorily reproduces the measurements obtained by [W. Nernst](#) and his pupils, but is also compatible with the elastic and optical properties of these substances. The quantum of action also comes to the fore in considering the specific heat of gases. W. Nernst had earlier suggested that to the quantum of energy of a vibration there must also correspond a quantum of energy of a rotation, and accordingly it was to be expected that the rotational energy of the gas molecules would disappear with falling temperature. The measurements by A. Eucken on the specific heat of hydrogen confirmed this conclusion, and if the calculations of A. Einstein and O. Stern, P. Ehrenfest and others have not until now afforded any completely satisfactory agreement, this lies understandably in our, as yet, incomplete knowledge of the model of a hydrogen molecule. The fact that the rotations of the gas molecules, as specified by quantum conditions, do really exist in Nature, can no longer be doubted in view of the work on absorption bands in the infrared by N. Bjerrum, E. von Bahr, H. Rubens, G. Hetmer and others, even though it has not been possible to give an all-round exhaustive explanation of this remarkable rotation spectra up to now.

Since, ultimately, all affinity properties of a substance are determined by its entropy, the quantum-theoretical calculation of the entropy opens up the way to all the problems of chemical relationships. The Nernst chemical constant, which O. Sackur calculated directly through a combinatorial method as applied to oscillators, is characteristic for the absolute value of the entropy of a gas. H. Tetrode, in close association with the data to be obtained by measurement, determined the difference in entropy values between vapour and solid state by studying an evaporation process.

Whilst in the cases so far considered, states of thermodynamic equilibrium are concerned, for which therefore the measurements can only yield statistically average values relating to many particles and lengthy periods of time, the observation of electron impacts leads directly to the dynamic details of the process under examination. Thus the determination of the so-called resonance potential carried out by J. Franck and G. Hertz, or that concerning the critical velocity is the minimum an electron must possess in order to cause emission of a light quantum or photon by impact with a neutral atom, supplied a method of measuring the quantum of action which was as direct as could be wished for. The experiments by D.L. Webster and E. Wagner and others resulted in the development of methods suitable for the [Röntgen](#) spectrum which also gave completely compatible results.

The production of photons by electron impact appears as the reverse process to that of electron emission through irradiation by light-, Röntgen-, or gamma-rays and again here, the energy quanta, determined by the quantum of action and by the vibration frequency, play a characteristic role, as could be recognized, already at an early time, from the striking fact that the velocity of the emitted electrons is not determined by the intensity of radiation, but only by the colour of the light incident upon the substance. Also from the quantitative aspect, Einstein's equations with respect to the light quantum have proved true in every way, as established by R.A. Millikan, in particular, by measurements of the escape velocity of emitted electrons, whilst the significance of the photon for the initiation of photochemical reactions was discovered by E. Warburg.

If the various experiments and experiences gathered together by me up to now, from the different fields of physics, provide impressive proof in favour of the existence of the quantum

of action, the quantum hypothesis has, nevertheless, its greatest support from the establishment and development of the atom theory by [Niels Bohr](#). For it fell to this theory to discover, in the quantum of action, the long-sought key to the entrance gate into the wonderland of spectroscopy, which since the discovery of spectral analysis had obstinately defied all efforts to breach it. And now that the way was opened, a sudden flood of new-won knowledge poured out over the whole field including the neighbouring fields in physics and chemistry. The first brilliant acquisition was the derivation of Balmer's series formula for hydrogen and helium including the reduction of the universal Rydberg constant to merely known numerical quantities, whereby even the small discrepancies for hydrogen and helium were recognized as essentially determined by the weak motion of the heavy atom nucleus. Investigation then turned to other series in the optical and the Röntgen spectrum using the extremely fruitful Ritz combination principle, which was at last revealed clearly in all its fundamental significance.

Whoever, in view of the numerous agreements which in the case of the special accuracy of spectroscopic measurements could lay claim to particularly striking confirmatory power, might have been still inclined to feel that it was all attributable to the play of chance, would be forced, finally, to discard even his last doubt, as A. Sommerfeld showed that from a logical extension of the laws of quantum distribution in systems with several degrees of freedom, and out of consideration of the variability of the inertial mass in accordance with the relativity theory, that magic formula arose before which both the hydrogen and the helium spectrum had to reveal the riddle of their fine structure, to such an extent that the finest present-day measurements, those of F. Paschen, could be explained generally through it - an achievement fully comparable with that of the famous discovery of the planet Neptune whose existence and orbit was calculated by Leverrier before the human eye had seen it. Progressing further along the same path, P. Epstein succeeded in fully explaining the Stark effect of the electrical splitting up of the spectral lines, P. Debye produced a simple explanation of the K-series of the Röntgen spectrum, which had been investigated by [Manne Siegbahn](#), and now followed a great number of further experiments, which illuminated with more or less success the dark secrets of the construction of the atom.

After all these results, towards whose complete establishment still many reputable names ought essentially to have been mentioned here, there is no other decision left for a critic who does not intend to resist the facts, than to award to the quantum of action, which by each different process in the colourful show of processes, has ever-again yielded the same result, namely, 6.52×10^{-27} erg sec, for its magnitude, full citizenship in the system of universal physical constants. It must certainly appear a unique coincidence that just in that time when the ideas of general relativity have broken through, and have led to fantastic results, Nature should have revealed an "absolute" in a place where it could be least expected, an invariable unit, in fact, by means of which the action quantity, contained in a space-time element, can be represented by a completely definite non-arbitrary number, and thereby divested itself of its (until now) relative character.

To be sure, the introduction of the quantum of action has not yet produced a genuine quantum theory. In fact, the path the research worker must yet tread to it is not less than that from the discovery of the velocity of light by Olaf Römer to the establishment of Maxwell's theory of light. The difficulties which the introduction of the quantum of action into the well-tried classical theory has posed right from the start have already been mentioned by me. During the course of the years they have increased rather than diminished, and if, in the meantime, the impetuous forward-driving research has passed to the order of the day for some of these,

temporarily, the gaps left behind, awaiting subsequent filling, react even harder upon the conscientious systematologist. What serves in Bohr's theory as a basis to build up the laws of action, is assembled out of specific hypotheses which, up to a generation ago, would undoubtedly have been flatly rejected altogether by every physicist. The fact that in the atom, certain quite definite quantum-selected orbits play a special role, might be taken still as acceptable, less easily however, that the electrons, circulating in these orbits with definite acceleration, radiate no energy at all. The fact that the quite sharply defined frequency of an emitted photon should be different from the frequency of the emitting electron must seem to a theoretical physicist, brought up in the classical school, at first sight to be a monstrous and, for the purpose of a mental picture, a practically intolerable demand.

But numbers decide, and the result is that the roles, compared with earlier times, have gradually changed. What initially was a problem of fitting a new and strange element, with more or less gentle pressure, into what was generally regarded as a fixed frame has become a question of coping with an intruder who, after appropriating an assured place, has gone over to the offensive; and today it has become obvious that the old framework must somehow or other be burst asunder. It is merely a question of where and to what degree. If one may make a conjecture about the expected escape from this tight corner, then one could remark that all the signs suggest that the main principles of thermodynamics from the classical theory will not only rule unchallenged but will more probably become correspondingly extended. What the armchair experiments meant for the foundation of classical thermodynamics, the adiabatic hypothesis of P. Ehrenfest means, provisionally, to the quantum theory; and in the same way as R. Clausius, as a starting point for the measurement of entropy, introduced the principle that, when treated appropriately, any two states of a material system can, by a reversible process, undergo a transition from one to the other, now the new ideas of Bohr's open up a very similar path into the interior of a wonderland hitherto hidden from him.

There is in particular one problem whose exhaustive solution could provide considerable elucidation. What becomes of the energy of a photon after complete emission? Does it spread out in all directions with further propagation in the sense of Huygens' wave theory, so constantly taking up more space, in boundless progressive attenuation? Or does it fly out like a projectile in one direction in the sense of Newton's emanation theory? In the first case, the quantum would no longer be in the position to concentrate energy upon a single point in space in such a way as to release an electron from its atomic bond, and in the second case, the main triumph of the Maxwell theory - the continuity between the static and the dynamic fields and, with it, the complete understanding we have enjoyed, until now, of the fully investigated interference phenomena - would have to be sacrificed, both being very unhappy consequences for today's theoreticians.

Be that as it may, in any case no doubt can arise that science will master the dilemma, serious as it is, and that which appears today so unsatisfactory will in fact eventually, seen from a higher vantage point, be distinguished by its special harmony and simplicity. Until this aim is achieved, the problem of the quantum of action will not cease to inspire research and fructify it, and the greater the difficulties which oppose its solution, the more significant it finally will show itself to be for the broadening and deepening of our whole knowledge in physics.

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